QUIZ ON CHAPTER 7 - SOLUTIONS

LOG AND EXPONENTIAL FUNCTIONS; MATH 150 – FALL 2016 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

1) Find the following derivatives. Simplify completely unless you are told not to. Do not use logarithmic differentiation unless you are told to. (60 points total)

a) $D_w\left(\ln\left[\sin(4w)\right]\right)$ (6 points)

$$= \frac{1}{\sin(4w)}\cdot D_w\left[\sin(4w)\right] = \frac{1}{\sin(4w)}\cdot\left[\cos(4w)\right]\cdot\left[4\right] = \boxed{4\cot(4w)}$$

b) $D_\theta\left(\log\left[\sec(\theta)\right]\right)$ (6 points)

$$= D_\theta\left(\log_{10}\left[\sec(\theta)\right]\right) = D_\theta\left[\frac{\ln\left[\sec(\theta)\right]}{\ln(10)}\right] \quad \text{(by Change-of-Base Property of Logs)}$$

$$= \frac{1}{\ln(10)}\cdot D_\theta\left(\ln\left[\sec(\theta)\right]\right) = \frac{1}{\ln(10)}\cdot\frac{1}{\sec(\theta)}\cdot D_\theta\left[\sec(\theta)\right]$$

$$= \frac{1}{\ln(10)}\cdot\frac{1}{\underset{(1)}{\cancel{\sec(\theta)}}}\cdot\left(\overset{(1)}{\cancel{\sec(\theta)}}\tan(\theta)\right) = \boxed{\frac{\tan(\theta)}{\ln(10)}}$$

c) $D_x\left[(2x+1)^9\cdot e^{7x}\right]$ (7 points)

You do not have to factor your answer.

$$= \left(D_x\left[(2x+1)^9\right]\right)\cdot\left[e^{7x}\right] + \left[(2x+1)^9\right]\cdot\left[D_x\left(e^{7x}\right)\right]$$

(by the Product Rule for Derivatives)

$$= \left(\left[9(2x+1)^8\right]\left[D_x(2x+1)\right]\right)\cdot\left[e^{7x}\right] + \left[(2x+1)^9\right]\cdot\left[7e^{7x}\right]$$

$$= \left(\left[9(2x+1)^8\right]\left[2\right]\right)\cdot\left[e^{7x}\right] + \left[(2x+1)^9\right]\cdot\left[7e^{7x}\right]$$

$$= \boxed{\begin{gathered}18(2x+1)^8 e^{7x} + 7(2x+1)^9 e^{7x},\ \text{or}\ (2x+1)^8 e^{7x}\left[18 + 7(2x+1)\right],\\ \text{or}\ (2x+1)^8 e^{7x}\left[18+14x+7\right],\ \text{or}\ (2x+1)^8(14x+25)e^{7x}\end{gathered}}$$

d) $D_r\left[7^{\csc(r)}\right]$ (6 points)

Answer only is fine, though logarithmic differentiation may help.

$$= \left[7^{\csc(r)}\ln(7)\right]\cdot\left(D_r\left[\csc(r)\right]\right) = \left[7^{\csc(r)}\ln(7)\right]\cdot\left[-\csc(r)\cot(r)\right]$$

$$= \boxed{-7^{\csc(r)}\ln(7)\csc(r)\cot(r)}$$

e) $D_x\left[\dfrac{(5x-7)^4(e^x)}{\sqrt[3]{x-1}}\right]$ (18 points)

You <u>must</u> use logarithmic differentiation and apply appropriate laws of logarithms whenever they apply, as in class. You do <u>not</u> have to write your final answer as a single fraction.

Let $y = \dfrac{(5x-7)^4(e^x)}{\sqrt[3]{x-1}} \Rightarrow$

$$\ln(y) = \ln\left[\frac{(5x-7)^4(e^x)}{\sqrt[3]{x-1}}\right] \quad \text{(We will expand the right side using Log Laws.)}$$

$$\ln(y) = \ln\left[(5x-7)^4\right] + \ln(e^x) - \ln\left(\sqrt[3]{x-1}\right) \quad \text{(by Product, Quotient Rules of Logs)}$$

$$\ln(y) = \ln\left[(5x-7)^4\right] + x - \ln\left[(x-1)^{1/3}\right] \quad \text{(by Inverse Properties of Logs)}$$

$$\ln(y) = 4\ln(5x-7) + x - \frac{1}{3}\ln(x-1) \quad \text{(by Power Rule of Logs)} \Rightarrow$$

(Note: Technically, we should write $\ln|y|$, etc., but don't worry.)

Use Implicit Differentiation to D_x both sides.

$$D_x\left[\ln(y)\right] = 4\cdot D_x\left[\ln(5x-7)\right] + D_x[x] - \frac{1}{3}\cdot D_x\left[\ln(x-1)\right]$$

$$\frac{1}{y}\cdot y' = 4\left(\frac{1}{5x-7}\cdot 5\right) + 1 - \frac{1}{3}\left(\frac{1}{x-1}\cdot 1\right)$$

$$\frac{y'}{y} = \frac{20}{5x-7} + 1 - \frac{1}{3(x-1)}$$

Multiply both sides by y, and expand y in terms of x.

$$y' = \left(\frac{20}{5x-7} + 1 - \frac{1}{3(x-1)}\right)y$$

$$\boxed{y' = \left(\frac{20}{5x-7} + 1 - \frac{1}{3(x-1)}\right)\left[\frac{(5x-7)^4(e^x)}{\sqrt[3]{x-1}}\right], \text{ or } \frac{e^x(5x-7)^3(15x^2+19x-32)}{3(x-1)^{4/3}}}$$

f) $D_x\left[x^{(\sqrt{x})}\right]$ (11 points)

You <u>must</u> use logarithmic differentiation.
You do <u>not</u> have to write your final answer as a single fraction.

We want to differentiate something of the form $\left[f(x)\right]^{g(x)}$, so let's use Logarithmic Differentiation to help bring that exponent down to earth!

Let $y = x^{(\sqrt{x})} \Rightarrow$

$$\ln(y) = \ln\left[x^{(\sqrt{x})}\right]$$

$$\ln(y) = \left(\sqrt{x}\right)\ln(x) \quad \left(\text{by Power Rule of Logs}\right) \Rightarrow$$

Now, use Implicit Differentiation to D_x both sides.

$$D_x\left[\ln(y)\right] = D_x\left[\left(\sqrt{x}\right)\ln(x)\right]$$

Now, use the Product Rule for Derivatives.

$$\frac{1}{y}\cdot y' = \left[D_x\left(\sqrt{x}\right)\right]\cdot\left[\ln(x)\right] + \left[\sqrt{x}\right]\cdot\left(D_x\left[\ln(x)\right]\right)$$

$$\frac{y'}{y} = \left[D_x\left(x^{1/2}\right)\right]\cdot\left[\ln(x)\right] + \left[\sqrt{x}\right]\cdot\left(D_x\left[\ln(x)\right]\right)$$

$$\frac{y'}{y} = \left[\frac{1}{2}x^{-1/2}\right]\cdot\left[\ln(x)\right] + \left[\sqrt{x}\right]\cdot\left[\frac{1}{x}\right]$$

$$\frac{y'}{y} = \frac{\ln(x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}}$$

Multiply both sides by y, and expand y in terms of x.

$$y' = \left[\frac{\ln(x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}}\right]y$$

$$y' = \boxed{\left[\frac{\ln(x)+2}{2\sqrt{x}}\right]\left[x^{(\sqrt{x})}\right], \quad \text{or} \quad \frac{x^{\left(\sqrt{x}-\frac{1}{2}\right)}\left[\ln(x)+2\right]}{2}}$$

g) $D_x\left[\tan^4\left(e^x\right)\right]$ (6 points)

$$= D_x\left(\left[\tan\left(e^x\right)\right]^4\right)$$

$$= 4\left[\tan\left(e^x\right)\right]^3\cdot D_x\left[\tan\left(e^x\right)\right] \quad \left(\text{by the Generalized Power Rule of Diff'n.}\right)$$

$$= 4\left[\tan\left(e^x\right)\right]^3\cdot\left[\sec^2\left(e^x\right)\right]\cdot D_x\left[e^x\right] = 4\left[\tan\left(e^x\right)\right]^3\cdot\left[\sec^2\left(e^x\right)\right]\cdot\left[e^x\right]$$

$$= \boxed{4e^x\tan^3\left(e^x\right)\sec^2\left(e^x\right)}$$

2) Evaluate the following integrals. Simplify completely. (45 points total)

a) $\displaystyle\int_{-4}^{-2} \frac{1}{4x+5}\, dx$ (10 points)

Give an exact answer; do not approximate.

The integrand is continuous on $[-4,-2]$. Using Guess-and-Check:

$$\int_{-4}^{-2} \frac{1}{4x+5}\, dx = \left[\frac{1}{4}\ln|4x+5|\right]_{-4}^{-2} = \left[\frac{1}{4}\ln|4(-2)+5|\right] - \left[\frac{1}{4}\ln|4(-4)+5|\right]$$

$$= \frac{1}{4}\ln|-3| - \frac{1}{4}\ln|-11| = \boxed{\frac{1}{4}\left[\ln(3)-\ln(11)\right], \text{ or } \frac{1}{4}\ln\left(\frac{3}{11}\right), \text{ or } \ln\left(\sqrt[4]{\frac{3}{11}}\right)}$$

Without using Guess-and-Check: $\left[\begin{array}{l} \text{Let } u = 4x+5 \Rightarrow \\ du = 4\,dx \Rightarrow \left(\text{Can use: } dx = \frac{1}{4}\,du\right) \end{array}\right]$

Change the limits of integration: $\left[\begin{array}{l} x=-4 \Rightarrow u = 4(-4)+5 = -11 \Rightarrow u=-11 \\ x=-2 \Rightarrow u = 4(-2)+5 = -3 \Rightarrow u=-3 \end{array}\right]$

$$\int_{-4}^{-2} \frac{1}{4x+5}\, dx = \frac{1}{4}\int_{-4}^{-2}\left[\frac{1}{4x+5}\right]\cdot 4\,dx \ (\text{Compensation}) = \frac{1}{4}\int_{-11}^{-3}\frac{du}{u} = \frac{1}{4}\Big[\ln|u|\Big]_{-11}^{-3}$$

$$= \frac{1}{4}\left(\ln|-3| - \ln|-11|\right) = \boxed{\frac{1}{4}\left[\ln(3)-\ln(11)\right], \text{ or } \frac{1}{4}\ln\left(\frac{3}{11}\right), \text{ or } \ln\left(\sqrt[4]{\frac{3}{11}}\right)}$$

b) $\displaystyle\int e^{2x}\sec\left(e^{2x}+1\right) dx$ (9 points)

$$\left[\begin{array}{l} \text{Let } u = e^{2x}+1 \Rightarrow \\ du = 2e^{2x}\,dx \Rightarrow \left(\text{Can use: } e^{2x}\,dx = \frac{1}{2}\,du\right) \end{array}\right]$$

$$\int e^{2x}\sec\left(e^{2x}+1\right) dx = \frac{1}{2}\int 2e^{2x}\sec\left(e^{2x}+1\right) dx \ (\text{Compensation})$$

$$= \frac{1}{2}\int\left[\sec\left(e^{2x}+1\right)\right]\cdot 2e^{2x}\,dx = \frac{1}{2}\int \sec(u)\,du = \frac{1}{2}\ln|\sec(u)+\tan(u)|+C$$

$$= \boxed{\frac{1}{2}\ln\left|\sec\left(e^{2x}+1\right)+\tan\left(e^{2x}+1\right)\right|+C, \text{ or } \ln\left(\sqrt{\left|\sec\left(e^{2x}+1\right)+\tan\left(e^{2x}+1\right)\right|}\right)+C}$$

c) $\displaystyle\int 8e^{-6t}\, dt$ (4 points)

$$\int 8e^{-6t}\,dt = 8\int e^{-6t}\,dt \ \left(\text{You can let: } u=-6t.\right)$$

$$= 8\left[\frac{e^{-6t}}{-6}\right]+C = \boxed{-\frac{4}{3}e^{-6t}+C, \text{ or } C-\frac{4}{3e^{6t}}}$$

(You can check that its derivative is the integrand.)

d) $\int \frac{2^{\ln(x)}}{x}\,dx$ (7 points)

$$u=\ln(x) \Rightarrow \qquad du=\frac{1}{x}\,dx$$

$$\int \frac{2^{\ln(x)}}{x}\,dx = \int \left[2^{\ln(x)}\right]\cdot\left[\frac{1}{x}\,dx\right] = \int 2^u\,du = \frac{2^u}{\ln(2)}+C = \boxed{\frac{2^{\ln(x)}}{\ln(2)}+C}$$

e) $\int \tan(x)\,dx$ (9 points)

You <u>must</u> show all work, as in class!

$$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx$$

$$= -\int \frac{-\sin(x)}{\cos(x)}\,dx \quad \text{(by Compensation)}$$

$$\left[\begin{array}{l} \text{Let } u=\cos(x) \Rightarrow \\ \quad du=-\sin(x)\,dx \Rightarrow \\ \left(\text{Can use: } \sin(x)\,dx=-du\right) \end{array}\right]$$

$$= -\int \frac{du}{u}$$

$$= -\ln|u|+C$$

$$= \boxed{\begin{array}{l} -\ln|\cos(x)|+C, \text{ or } \ln\left(|\cos(x)|^{-1}\right)+C, \\ \quad \text{or } \ln|\sec(x)|+C \end{array}}$$

f) $\int \csc(5x)\,dx$ (6 points)

Answer only is fine.

$$\int \csc(5x)\,dx = \boxed{\frac{1}{5}\ln|\csc(5x)-\cot(5x)|+C}$$

<u>Showing the u-sub (Optional):</u>

$$u=5x \Rightarrow$$

$$du=5\,dx \Rightarrow \left(\text{Can use: } dx=\frac{1}{5}\,du\right)$$

$$\int \csc(5x)\,dx = \frac{1}{5}\int\left[5\csc(5x)\right]dx \quad \text{(Compensation)}$$

$$= \frac{1}{5}\int\left[\csc(5x)\right]\cdot\left[5\,dx\right]$$

$$= \frac{1}{5}\int \csc(u)\,du$$

$$= \frac{1}{5}\ln|\csc(u)-\cot(u)|+C$$

$$= \boxed{\frac{1}{5}\ln|\csc(5x)-\cot(5x)|+C}$$