MATH 150: SOME BASIC TRIG - KNOW THIS!
(SECTION 1.3; back endpapers)

FUNDAMENTAL TRIG IDENTITIES (IDs)

Memorize these in both "directions" (i.e., left-to-right and right-to-left).

Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \qquad \sin x = \frac{1}{\csc x}$$

$$\sec x = \frac{1}{\cos x} \qquad \cos x = \frac{1}{\sec x}$$

$$\cot x = \frac{1}{\tan x} \qquad \tan x = \frac{1}{\cot x}$$

Warning: Remember that the reciprocal of $\sin x$ is $\csc x$, not $\sec x$.

Note: We typically treat "0" and "undefined" as reciprocals when we are dealing with trig functions. Your algebra teacher will not want to hear this, though!

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \text{and} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$\tan^2 x + 1 = \sec^2 x$$

Tip: The 2nd and 3rd IDs can be obtained by dividing both sides of the 1st ID by $\sin^2 x$ and $\cos^2 x$, respectively.

Tip: The squares of $\csc x$ and $\sec x$, which have the "Up-U, Down-U" graphs, are all alone on the right sides of the last two IDs. They can never be 0 in value. (Why is that? Look at the left sides.)

Cofunction Identities

If x is measured in radians, then:

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$
$$\cos x = \sin\left(\frac{\pi}{2} - x\right)$$

We have analogous relationships for tan and cot, and for sec and csc; remember that they are sometimes undefined.

Think: Cofunctions of complementary angles are equal.

Even / Odd (or Negative Angle) Identities

Among the six basic trig functions, cos (and its reciprocal, sec) are even:

$$\cos(-x) = \cos x$$
$$\sec(-x) = \sec x, \text{ when both sides are defined}$$

However, the other four (sin and csc, tan and cot) are odd:

$$\sin(-x) = -\sin x$$
$$\csc(-x) = -\csc x, \text{ when both sides are defined}$$

$$\tan(-x) = -\tan x, \text{ when both sides are defined}$$
$$\cot(-x) = -\cot x, \text{ when both sides are defined}$$

Note: If f is an even function (such as cos), then the graph of $y = f(x)$ is symmetric about the y-axis.

Note: If f is an odd function (such as sin), then the graph of $y = f(x)$ is symmetric about the origin.