SOLUTIONS TO THE FINAL

MATH 151 – FALL 2003 – KUNIYUKI
45 POINTS TOTAL; 3 POINTS FOR EACH PROBLEM
An appropriate sheet of notes is allowed.

Give the best answers based on the notes and our discussions in class.

1) We would use a u-substitution to evaluate $\int \tan^5 x \sec^5 x \, dx$. What would be our choice for u?

   We like the fact that the tan power is odd.
   We will let u be the "other guy," namely $\mathbf{\sec x}$.

2) We would use a trig substitution to evaluate $\int \frac{1}{x^3 \sqrt{9x^2 - 25}} \, dx$. What would we use as our trig substitution?

   The radical is of the form $\sqrt{u^2 - a^2}$, where $u = 3x$ and $a = 5$.
   Our substitution should be: $\mathbf{3x = 5\sec\theta}$ or $\mathbf{x = \frac{5}{3}\sec\theta}$

3) We want to integrate $\int \frac{1}{x^2 (x-3)(x^2+16)} \, dx$ using partial fractions.

   Write the <u>form</u> of the partial fraction decomposition for the integrand, $\frac{1}{x^2 (x-3)(x^2+16)}$.

   $$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3} + \frac{Dx+E}{x^2+16}$$

4) Find $\lim_{x \to 0} \frac{4x}{\tan x}$. Write ∞ or $-\infty$ if appropriate. If the limit does not exist, and ∞ and $-\infty$ are inappropriate, write "DNE" (Does Not Exist).

   $$\lim_{x \to 0} \frac{4x}{\tan x} \quad \frac{\to 0}{\to \tan 0 = 0} \quad \left(\frac{0}{0}\right)$$

   $$\overset{\text{L'H}}{=} \lim_{x \to 0} \frac{4}{\sec^2 x} \quad \frac{\to 4}{\to \sec^2 0 = (1)^2 = 1}$$

   $$= \mathbf{4}$$

5) True or False: Both 0^0 and 1^∞ are indeterminate limit forms. Circle one:

   True False

6) Fill in the boxes:

$$\int_{-3}^{5} \frac{1}{(x+1)^5}\,dx = \int_{-3}^{-1} \frac{1}{(x+1)^5}\,dx + \int_{-1}^{5} \frac{1}{(x+1)^5}\,dx$$

$$= \lim_{t\to -1^-} \int_{-3}^{t} \frac{1}{(x+1)^5}\,dx + \lim_{w\to -1^+} \int_{w}^{5} \frac{1}{(x+1)^5}\,dx$$

7) Find the sum of the geometric series $\sum_{n=1}^{\infty} 3\left(\frac{1}{4}\right)^{n-1}$.

$$S = \frac{a_1}{1-r} = \frac{3}{1-\frac{1}{4}} = \frac{3}{\frac{3}{4}} = 3\cdot\frac{4}{3} = \mathbf{4}$$

8) When using the Integral Test, we use an interpolating function $f(x)$ to analyze the series $\sum a_n$. For example, we use $f(x) = \frac{1}{x^2}$ to analyze $\sum \frac{1}{n^2}$. State the assumptions (hypotheses) that we require of f if we are going to apply the Integral Test.

f must be positive-valued, continuous, and decreasing on the relevant interval.

9) The series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^{3/4}}$ is … (circle one)

Absolutely Convergent **Conditionally Convergent** Divergent

The series converges by the AST (Alternating Series Test), but the "absolute value" series $\sum_{n=1}^{\infty} \frac{1}{n^{3/4}}$ diverges by the *p*-series Test.

10) Write the first four nonzero terms of the Maclaurin series for $f(x) = \cos x$.

$$\mathbf{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots}$$

11) True or False: A Maclaurin series $\sum_{n=0}^{\infty} a_n x^n$ can have $[-2,5)$ as its interval of convergence. Circle one:

True **False**

Since the Maclaurin series (which is a power series) is centered at 0, the interval of convergence must also be centered at 0. How else could we talk about the "radius of convergence?"

12) Write the form for the Taylor series representation of $f(x)$ centered at c, assuming it exists.

$$\sum_{n=0}^{\infty} \frac{f^{n}(c)}{n!}(x-c)^{n}$$

13) $x=\cos^2 t, y=\sec^4 t$. Eliminate the parameter to get an equation in x and y.

$\sec^4 t = \frac{1}{(\cos^2 t)^2}$, so $\mathbf{y = \frac{1}{x^2}}$.

14) The graph of the polar equation $\theta = 4$ is a … (circle one).

Circle **Line** Neither

15) Write the rectangular equation $x^2 + y^2 = 4y$ as a polar equation in r and θ.

$\mathbf{r^2 = 4r\sin\theta}$