

MATH 151 POP QUIZ: SOLUTIONS

DERIVATIVES (CHAPTERS 3, 7, and 8)

$$D_x(\sin x) = \underline{\cos x}$$

$$D_x(\cos x) = \underline{-\sin x}$$

$$D_x(\tan x) = \underline{\sec^2 x}$$

$$D_x(\cot x) = \underline{-\csc^2 x}$$

$$D_x(\sec x) = \underline{\sec x \tan x}$$

$$D_x(\csc x) = \underline{-\csc x \cot x}$$

Take a look at each row: we are taking the derivatives of a pair of cofunctions. To get from the first column to the second column, you flip the sign of the result and take the cofunction of each factor.

Evaluate the following:

$$D_x\left(4x^3 - \sqrt[3]{x} + \frac{1}{x^5} + 7\right)$$

D_x is a linear operator: we can differentiate term-by-term, and constant factors “pop out.”

$$\begin{aligned} & D_x(4x^3 - x^{1/3} + x^{-5} + 7) \\ &= 4(3x^2) - \frac{1}{3}x^{-2/3} - 5x^{-6} + 0 \quad (\text{Power Rule: } D_x(x^n) = nx^{n-1}) \\ &= 12x^2 - \frac{1}{3}x^{-2/3} - 5x^{-6} \end{aligned}$$

$$\begin{aligned} D_x(7x^2 + 3)^{10} &= 10(7x^2 + 3)^9 \cdot \underbrace{D_x(7x^2 + 3)}_{\text{"Tail" from the Chain Rule}} \\ &= 10(7x^2 + 3)^9 \cdot 14x \\ &= 140x(7x^2 + 3)^9 \end{aligned}$$

$$\begin{aligned} D_x[\sec(4x)] &= \sec(4x)\tan(4x) \cdot \underbrace{D_x(4x)}_{\text{Tail} = 4} \\ &= 4 \sec(4x)\tan(4x) \end{aligned}$$

$$\begin{aligned}
D_x[\sin^4(3x)] &= D_x[\sin(3x)]^4 \\
&= 4[\sin(3x)]^3 \cdot \underbrace{D_x[\sin(3x)]}_{\text{Tail}} \\
&= 4\sin^3(3x) \cdot \cos(3x) \cdot \underbrace{D_x(3x)}_{=3} \\
&= 12\sin^3(3x)\cos(3x)
\end{aligned}$$

$$\begin{aligned}
D_x(x^3 \tan x) &= [D_x(x^3)][\tan x] + [x^3][D_x(\tan x)] \quad (\text{by the Product Rule}) \\
&= 3x^2 \tan x + x^3 \sec^2 x
\end{aligned}$$

$$\begin{aligned}
D_x\left(\frac{\sin x}{3x+1}\right) &= \frac{\text{Lo} \cdot \text{D(Hi)} - \text{Hi} \cdot \text{D(Lo)}}{\text{the square of what's below}} \quad (\text{by the Quotient Rule}) \\
&= \frac{(3x+1) \cdot D_x(\sin x) - (\sin x) \cdot \overbrace{D_x(3x+1)}^{=3}}{(3x+1)^2} \\
&= \frac{(3x+1)\cos x - 3\sin x}{(3x+1)^2}
\end{aligned}$$

$$D_x(e^x) = e^x$$

$$\begin{aligned}
D_x(e^{4x}) &= e^{4x} \cdot \underbrace{D_x(4x)}_{\text{Tail}=4} \quad \left(\text{Think: } D_x[e^{\text{blah}}] = e^{\text{blah}} \cdot D_x(\text{blah}).\right) \\
&= 4e^{4x}
\end{aligned}$$

$$D_x(e^{\sin x}) = e^{\sin x} \underbrace{\cos x}_{\text{Tail}}$$

$$D_x(2^x) = 2^x \ln 2$$

$$D_x(\ln x) = \frac{1}{x}$$

$$\begin{aligned}
D_x[\ln(x^3 + 4x)] &= \frac{1}{x^3 + 4x} \cdot \overbrace{D_x(x^3 + 4x)}^{\text{Tail} = 3x^2 + 4} \quad \left(\text{Think: } D_x[\ln(\text{blah})] = \frac{1}{\text{blah}} \cdot D_x(\text{blah}).\right) \\
&= \frac{3x^2 + 4}{x^3 + 4x}
\end{aligned}$$

$$D_x(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$D_x(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$D_x(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$D_x(\sinh x) = \cosh x$$

$$D_x(\cosh x) = \sinh x$$

Let $f(x) = x^2$. Give a geometric interpretation of $f'(3)$.

$f'(3)$ is the slope of the tangent line to the graph of f at the point $(3,9)$.

$f'(x) = 2x$, so $f'(3) = 2(3) = 6$.

INTEGRALS (CHAPTERS 5, 7, and 8)

You can often check to see if the derivative of your answer is the original integrand.

\int is a linear operator: we can integrate term-by-term, and constant factors “pop out.”

Evaluate the following:

$$\begin{aligned} \int \left(5x^2 + \frac{1}{x^4} - \sqrt{x} \right) dx &= \int \left(5x^2 + x^{-4} - x^{1/2} \right) dx \left(\text{Power Rule: } \int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1 \right) \\ &= 5 \left(\frac{x^3}{3} \right) + \frac{x^{-3}}{-3} - \frac{x^{3/2}}{3/2} + C \\ &= \frac{5}{3}x^3 - \frac{1}{3}x^{-3} - \frac{2}{3}x^{3/2} + C \end{aligned}$$

Remember that you need a constant of integration (+ C) when you are evaluating an indefinite integral.

$$\int \frac{x^2}{x^3+7} dx = \frac{1}{3} \int \frac{3x^2}{x^3+7} dx$$

Let $u = x^3 + 7$. (We often let u be an "inside guy" or a denominator.)

$$du = 3x^2 dx. \text{ (The derivative of } u \text{ is a factor of the integrand.)}$$

We can insert a factor of "3" in the integrand if we compensate for it using $\frac{1}{3}$.

$$= \frac{1}{3} \int \frac{du}{u} \quad \left(\text{Also, } du = 3x^2 dx \Rightarrow x^2 dx = \frac{1}{3} du \right)$$

$$= \frac{1}{3} \ln |u| + C$$

$$= \frac{1}{3} \ln |x^3 + 7| + C$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C \quad (\text{Remember the inverse trig derivative formulas!})$$

$$\int \sin(6x) dx = -\frac{1}{6} \cos(6x) + C$$

You could use "Trial-and-Error," or "Guess-and-Check."

Take a guess at the correct answer and differentiate your guess. If you're off by a constant factor, throw in an appropriate "fudge factor."

$$\int \sec^2 x dx = \tan x + C \quad (\text{Remember the trig derivative formulas!})$$

$$\int_4^9 \underbrace{\frac{e^{\sqrt{x}}}{\sqrt{x}}}_{\substack{\text{Continuous} \\ \text{on } [4,9]}} dx = 2 \int_4^9 \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx$$

$$\text{Let } u = \sqrt{x} = x^{\frac{1}{2}}.$$

$$du = \frac{1}{2} x^{-\frac{1}{2}} dx = \frac{1}{2\sqrt{x}} dx.$$

Change the limits of integration! (Otherwise, go back to x at the end.)

$$x = 4 \rightarrow u = \sqrt{4} = 2$$

$$x = 9 \rightarrow u = \sqrt{9} = 3$$

$$= 2 \int_{u=2}^{u=3} e^u du$$

$$= 2[e^u]_2^3$$

$$= 2(e^3 - e^2)$$

The value of a definite integral (such as the one given here) is typically a number.

$$\int e^{5x} dx = \frac{1}{5} e^{5x} + C \quad (\text{You could use "Guess-and-Check."})$$

$$\int 2^x dx = \frac{2^x}{\ln 2} + C$$

$$\int \sin x dx = \underline{-\cos x + C}$$

$$\int \cos x dx = \underline{\sin x + C}$$

$$\int \tan x dx = \underline{\begin{array}{l} -\ln |\cos x| + C, \\ \text{or } \ln |\sec x| + C \end{array}}$$

$$\int \cot x dx = \underline{\ln |\sin x| + C}$$

$$\int \tan x dx = \int \frac{\sin x}{\cos x} dx$$

$$\text{Let } u = \cos x.$$

$$du = -\sin x dx.$$

$$= -\int \frac{du}{u}$$

$$= -\ln |u| + C$$

$$= -\ln |\cos x| + C, \text{ or}$$

$$\ln |\cos x|^{-1} + C, \text{ or}$$

$$\ln |\sec x| + C$$

$$\int \cot x dx = \int \frac{\cos x}{\sin x} dx$$

$$\text{Let } u = \sin x.$$

$$du = \cos x dx.$$

$$= \int \frac{du}{u}$$

$$= \ln |u| + C$$

$$= \ln |\sin x| + C$$

$$\int \sec x dx = \underline{\ln |\sec x + \tan x| + C}$$

$$\int \csc x dx = \underline{\ln |\csc x - \cot x| + C}$$

$$\int \sec x dx = \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} dx$$

$$= \int \frac{\overbrace{(\sec^2 x + \sec x \tan x)}^{du}}{\underbrace{\sec x + \tan x}_{\text{Let this be } u}} dx$$

$$= \int \frac{du}{u}$$

$$= \ln |u| + C$$

$$= \ln |\sec x + \tan x| + C$$

$$\int \csc x dx = \int \csc x \cdot \frac{\csc x - \cot x}{\csc x - \cot x} dx$$

$$= \int \frac{\overbrace{(\csc^2 x - \csc x \cot x)}^{du}}{\underbrace{\csc x - \cot x}_{\text{Let this be } u}} dx$$

$$= \int \frac{du}{u}$$

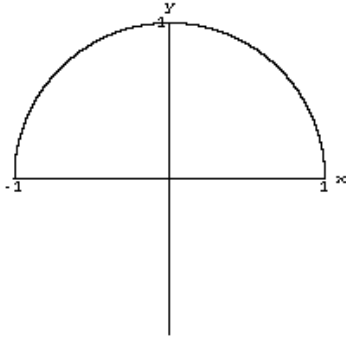
$$= \ln |u| + C$$

$$= \ln |\csc x - \cot x| + C$$

Give a geometric interpretation of $\int_{-1}^1 \sqrt{1-x^2} dx$.

$f(x) = \sqrt{1-x^2}$ represents the upper half of the unit circle [centered at the origin].

We can interpret definite integrals in terms of signed areas (areas above the x -axis yield positive contributions; areas below the x -axis yield negative contributions).



$$\int_{-1}^1 \sqrt{1-x^2} dx = \text{Area of semicircle} = \frac{1}{2} \pi r^2 = \frac{1}{2} \pi (1)^2 = \frac{\pi}{2}.$$

True or False: If $x > 0$ and $y > 0$, $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.

True or False: If $x > 0$, $\ln x^3 = 3 \ln x$.

True or False: If $x > 0$, $(\ln x)^3 = 3 \ln x$.

The front endpaper of your textbook has a nice list of key formulas. You should have learned all of these in Math 150, except for #1, #19, and #20 under FORMULAS FOR INTEGRALS.