MATH 151 POP QUIZ: SOLUTIONS

DERIVATIVES (CHAPTERS 3, 7, and 8)

$D_x(\sin x) = \underline{\quad \cos x \quad}$ $\qquad$ $D_x(\cos x) = \underline{\quad -\sin x \quad}$

$D_x(\tan x) = \underline{\quad \sec^2 x \quad}$ $\qquad$ $D_x(\cot x) = \underline{\quad -\csc^2 x \quad}$

$D_x(\sec x) = \underline{\quad \sec x \tan x \quad}$ $\qquad$ $D_x(\csc x) = \underline{\quad -\csc x \cot x \quad}$

Take a look at each row: we are taking the derivatives of a pair of cofunctions. To get from the first column to the second column, you flip the sign of the result and take the cofunction of each factor.

Evaluate the following:

$$D_x\left(4x^3 - \sqrt[3]{x} + \frac{1}{x^5} + 7\right)$$

D_x is a linear operator: we can differentiate term-by-term, and constant factors "pop out."

$$D_x\left(4x^3 - x^{1/3} + x^{-5} + 7\right)$$
$$= 4\left(3x^2\right) - \frac{1}{3}x^{-2/3} - 5x^{-6} + 0 \quad \left(\text{Power Rule: } D_x\left(x^n\right) = nx^{n-1}\right)$$
$$= 12x^2 - \frac{1}{3}x^{-2/3} - 5x^{-6}$$

$$D_x\left(7x^2+3\right)^{10} = 10\left(7x^2+3\right)^9 \cdot \underbrace{D_x\left(7x^2+3\right)}_{\text{"Tail" from the Chain Rule}}$$
$$= 10\left(7x^2+3\right)^9 \cdot 14x$$
$$= 140x\left(7x^2+3\right)^9$$

$$D_x\left[\sec(4x)\right] = \sec(4x)\tan(4x) \cdot \underbrace{D_x(4x)}_{\text{Tail} = 4}$$
$$= 4\sec(4x)\tan(4x)$$

$$D_x\left[\sin^4(3x)\right] = D_x\left[\sin(3x)\right]^4$$
$$= 4\left[\sin(3x)\right]^3 \cdot \underbrace{D_x\left[\sin(3x)\right]}_{\text{Tail}}$$
$$= 4\sin^3(3x)\cdot\cos(3x)\cdot\underbrace{D_x(3x)}_{=3}$$
$$= 12\,\sin^3(3x)\cos(3x)$$

$$D_x\left(x^3\tan x\right) = \left[D_x\left(x^3\right)\right]\left[\tan x\right] + \left[x^3\right]\left[D_x(\tan x)\right] \qquad \text{(by the Product Rule)}$$
$$= 3x^2\tan x + x^3\sec^2 x$$

$$D_x\left(\frac{\sin x}{3x+1}\right) = \frac{\text{Lo}\cdot\text{D(Hi)} - \text{Hi}\cdot\text{D(Lo)}}{\text{the square of what's below}} \qquad \text{(by the Quotient Rule)}$$
$$= \frac{(3x+1)\cdot D_x(\sin x) - (\sin x)\cdot\overbrace{D_x(3x+1)}^{=3}}{(3x+1)^2}$$
$$= \frac{(3x+1)\cos x - 3\sin x}{(3x+1)^2}$$

$$D_x\left(e^x\right) = e^x$$

$$D_x\left(e^{4x}\right) = e^{4x}\cdot\underbrace{D_x(4x)}_{\text{Tail}=4} \qquad \left(\text{Think: } D_x\left[e^{blah}\right] = e^{blah}\cdot D_x(blah).\right)$$
$$= 4e^{4x}$$

$$D_x\left(e^{\sin x}\right) = e^{\sin x}\underbrace{\cos x}_{\text{Tail}}$$

$$D_x\left(2^x\right) = 2^x\ln 2$$

$$D_x(\ln x) = \frac{1}{x}$$

$$D_x\left[\ln\left(x^3+4x\right)\right] = \frac{1}{x^3+4x}\cdot\overbrace{D_x\left(x^3+4x\right)}^{\text{Tail } = \, 3x^2+4} \qquad \left(\text{Think: } D_x\left[\ln(blah)\right] = \frac{1}{blah}\cdot D_x(blah).\right)$$
$$= \frac{3x^2+4}{x^3+4x}$$

$$D_x\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$D_x\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}$$

$$D_x\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

$$D_x(\sinh x) = \cosh x$$

$$D_x(\cosh x) = \sinh x$$

Let $f(x) = x^2$. Give a geometric interpretation of $f'(3)$.

$f'(3)$ is the slope of the tangent line to the graph of f at the point $(3,9)$.
$f'(x) = 2x$, so $f'(3) = 2(3) = 6$.

INTEGRALS (CHAPTERS 5, 7, and 8)

You can often check to see if the derivative of your answer is the original integrand.
$\int$ is a linear operator: we can integrate term-by-term, and constant factors "pop out."

Evaluate the following:

$$\int \left(5x^2 + \frac{1}{x^4} - \sqrt{x}\right) dx = \int \left(5x^2 + x^{-4} - x^{1/2}\right) dx \left(\text{Power Rule: } \int x^n \, dx = \frac{x^{n+1}}{n+1},\ n \neq -1\right)$$

$$= 5\left(\frac{x^3}{3}\right) + \frac{x^{-3}}{-3} - \frac{x^{3/2}}{3/2} + C$$

$$= \frac{5}{3}x^3 - \frac{1}{3}x^{-3} - \frac{2}{3}x^{3/2} + C$$

Remember that you need a constant of integration (+ C) when you are evaluating an indefinite integral.

$$\int \frac{x^2}{x^3+7}\,dx = \frac{1}{3}\int \frac{3x^2}{x^3+7}\,dx$$

Let $u = x^3 + 7$. (We often let u be an "inside guy" or a denominator.)

$du = 3x^2\,dx$. (The derivative of u is a factor of the integrand.)

We can insert a factor of "3" in the integrand if we compensate for it using $\frac{1}{3}$.

$$= \frac{1}{3}\int \frac{du}{u} \quad \left(\text{Also, } du = 3x^2\,dx \quad \Rightarrow \quad x^2\,dx = \frac{1}{3}du\right)$$

$$= \frac{1}{3}\ln|u| + C$$

$$= \frac{1}{3}\ln\left|x^3+7\right| + C$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$ (Remember the inverse trig derivative formulas!)

$$\int \sin(6x)\,dx = -\frac{1}{6}\cos(6x) + C$$

You could use “Trial-and-Error,” or “Guess-and-Check.”
Take a guess at the correct answer and differentiate your guess. If you’re off by a constant factor, throw in an appropriate “fudge factor.”

$$\int \sec^2 x\,dx = \tan x + C$$ (Remember the trig derivative formulas!)

$$\int_4^9 \underbrace{\frac{e^{\sqrt{x}}}{\sqrt{x}}}_{\substack{\text{Continuous}\\ \text{on } [4,9]}}\,dx = 2\int_4^9 \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$$

Let $u = \sqrt{x} = x^{\frac{1}{2}}$.

$$du = \frac{1}{2}x^{-\frac{1}{2}}\,dx = \frac{1}{2\sqrt{x}}\,dx.$$

Change the limits of integration! (Otherwise, go back to x at the end.)

$x = 4 \rightarrow u = \sqrt{4} = 2$

$x = 9 \rightarrow u = \sqrt{9} = 3$

$$= 2\int_{u=2}^{u=3} e^u\,du$$

$$= 2\left[e^u\right]_2^3$$

$$= 2\left(e^3 - e^2\right)$$

The value of a definite integral (such as the one given here) is typically a number.

$$\int e^{5x}\,dx = \frac{1}{5}e^{5x} + C$$ (You could use "Guess-and-Check.")

$$\int 2^x\,dx = \frac{2^x}{\ln 2} + C$$

$$\int \sin x\,dx = \underline{\quad -\cos x + C \quad}$$

$$\int \cos x\,dx = \underline{\quad \sin x + C \quad}$$

$$\int \tan x\,dx = -\ln|\cos x| + C, \text{ or } \ln|\sec x| + C$$

$$\int \cot x\,dx = \underline{\quad \ln|\sin x| + C \quad}$$

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$

Let $u = \cos x$.

$du = -\sin x\,dx$.

$$= -\int \frac{du}{u}$$

$$= -\ln|u| + C$$

$$= -\ln|\cos x| + C, \text{ or}$$

$$\ln|\cos x|^{-1} + C, \text{ or}$$

$$\ln|\sec x| + C$$

$$\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$$

Let $u = \sin x$.

$du = \cos x\,dx$.

$$= \int \frac{du}{u}$$

$$= \ln|u| + C$$

$$= \ln|\sin x| + C$$

$$\int \sec x\,dx = \underline{\quad \ln|\sec x + \tan x| + C \quad}$$

$$\int \csc x\,dx = \underline{\quad \ln|\csc x - \cot x| + C \quad}$$

$$\int \sec x\,dx = \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x}\,dx$$

$$= \int \frac{\overbrace{\left(\sec^2 x + \sec x \tan x\right)dx}^{du}}{\underbrace{\sec x + \tan x}_{\text{Let this be } u.}}$$

$$= \int \frac{du}{u}$$

$$= \ln|u| + C$$

$$= \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = \int \csc x \cdot \frac{\csc x - \cot x}{\csc x - \cot x}\,dx$$

$$= \int \frac{\overbrace{\left(\csc^2 x - \csc x \cot x\right)dx}^{du}}{\underbrace{\csc x - \cot x}_{\text{Let this be } u.}}$$

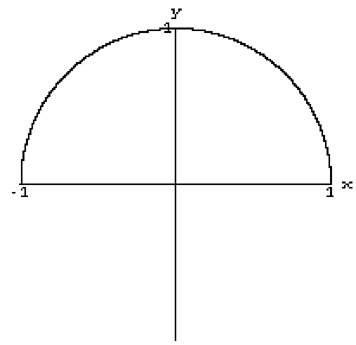
$$= \int \frac{du}{u}$$

$$= \ln|u| + C$$

$$= \ln|\csc x - \cot x| + C$$

Give a geometric interpretation of $\int_{-1}^{1} \sqrt{1-x^2}\,dx$.

$f(x) = \sqrt{1-x^2}$ represents the upper half of the unit circle [centered at the origin]. We can interpret definite integrals in terms of signed areas (areas above the x-axis yield positive contributions; areas below the x-axis yield negative contributions).



$\int_{-1}^{1} \sqrt{1-x^2}\,dx$ = Area of semicircle = $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi (1)^2 = \frac{\pi}{2}$.

True or **False**: If $x > 0$ and $y > 0$, $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.

True or False: If $x > 0$, $\ln x^3 = 3 \ln x$.

True or **False**: If $x > 0$, $(\ln x)^3 = 3 \ln x$.

The front endpaper of your textbook has a nice list of key formulas. You should have learned all of these in Math 150, except for #1, #19, and #20 under FORMULAS FOR INTEGRALS.