FINAL

The final will be scored out of 175 points (175 points = 100%), and then your score will be renormalized so that the total possible on the final is 300 points.

Show all work where appropriate!

1) Let p and q be propositions. Write the truth table for the proposition $\left[\neg(p \rightarrow q)\right] \land q$. (15 points)

2) Let the universe of discourse for x, y, and z be the set of <u>all rational</u> <u>numbers</u>. For each of the propositions below, indicate whether it is True (T) or False (F). (10 points total; 5 points each)

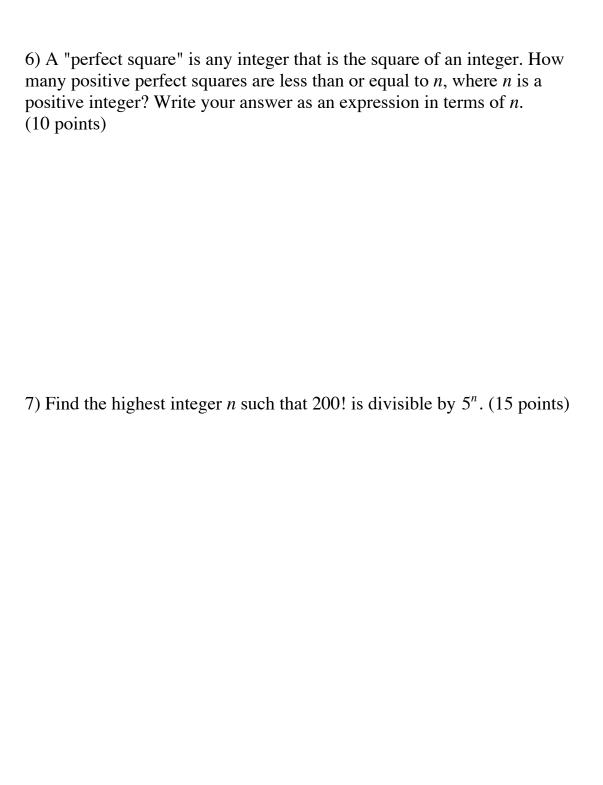
a)
$$\exists x \forall y \left(\frac{x}{y} = 1\right)$$

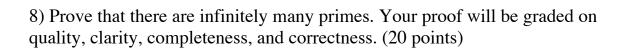
b)
$$\forall x \forall y \exists z (xy = z)$$

3) Let A and B be sets. Completely simplify $\overline{A \cup (B \cap \overline{B})} \cup \overline{A}$. (15 points)

4) Completely evaluate the sum $\sum_{i=1}^{500} 3i$. Do <u>not</u> use "brute force". (15 points)

- 5) Let the function $f: \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = 2^x$. (10 points; 5 points each)
 - a) Yes or No: Is f one-to-one?
 - b) Yes or No: Is f onto?





9) Use weak induction to prove

$$1+5+9+...+(4n-3)=n(2n-1)$$

for all positive integers n. Make sure your inductive proof is complete. (20 points)

10) Write an expression for the number of decimal strings of length 12 that have exactly 4 or exactly 5 zeros. You do not have to evaluate your expression completely; for example, you do not have to evaluate anything of the form $\binom{n}{r}$. (15 points)

11) How many different possible functions are there from a set of n elements $(n \in \mathbb{Z}^+)$ to a set of 7 elements? Write your answer as an expression in terms of n. (10 points)

Use an iterative approach to find the solution of the recurrence

relation
$$a_n = 1.5a_{n-1} + 3$$
, subject to the initial condition $a_0 = 5$.
Use the fact that $\sum_{i=0}^{k} r^i = \frac{r^{k+1} - 1}{r - 1}$ $(r \ne 1, k \text{ is a nonnegative integer})$.

(20 points)