Math 245 Name: ________________________

# FINAL

The final will be scored out of 175 points (175 points = 100%), and then your score will be renormalized so that the total possible on the final is 300 points.

Show all work where appropriate!

1) Let $p$ and $q$ be propositions. Write the truth table for the proposition $\left[\neg(p \to q)\right] \wedge q$. (15 points)

2) Let the universe of discourse for $x$, $y$, and $z$ be the set of <u>all rational numbers</u>. For each of the propositions below, indicate whether it is True (T) or False (F). (10 points total; 5 points each)

a) $\exists x \forall y \left(\frac{x}{y} = 1\right)$ __________

b) $\forall x \forall y \exists z \,(xy = z)$ __________

3) Let *A* and *B* be sets. Completely simplify $\overline{A \cup \left(B \cap \overline{B}\right)} \cup \overline{A}$. (15 points)

4) Completely evaluate the sum $\sum_{i=1}^{500} 3i$. Do <u>not</u> use "brute force". (15 points)

5) Let the function *f*: **R** → **R** be defined by $f(x) = 2^x$.
(10 points; 5 points each)

a) Yes or No: Is *f* one-to-one? ___________

b) Yes or No: Is *f* onto? ___________

6) A "perfect square" is any integer that is the square of an integer. How many positive perfect squares are less than or equal to *n*, where *n* is a positive integer? Write your answer as an expression in terms of *n*.
(10 points)

7) Find the highest integer *n* such that 200! is divisible by $5^n$. (15 points)

8) Prove that there are infinitely many primes. Your proof will be graded on quality, clarity, completeness, and correctness. (20 points)

9) Use weak induction to prove

$$1 + 5 + 9 + ... + (4n - 3) = n(2n - 1)$$

for all positive integers $n$. Make sure your inductive proof is complete. (20 points)

10) Write an expression for the number of decimal strings of length 12 that have exactly 4 or exactly 5 zeros. You do not have to evaluate your expression completely; for example, you do not have to evaluate anything of the form $\binom{n}{r}$. (15 points)

11) How many different possible functions are there from a set of $n$ elements $\left(n \in \mathbf{Z}^{+}\right)$ to a set of 7 elements? Write your answer as an expression in terms of $n$. (10 points)

12) Use an iterative approach to find the solution of the recurrence relation $a_n = 1.5a_{n-1} + 3$, subject to the initial condition $a_0 = 5$.

Use the fact that $\sum_{i=0}^{k} r^i = \frac{r^{k+1} - 1}{r - 1}$ $\left(r \neq 1, k \text{ is a nonnegative integer}\right)$.

(20 points)