Math 245 Name: ________________________

<u>QUIZ 1</u>

SECTIONS 1.1-1.3: LOGIC

1) For each of the following, indicate whether the equivalent "if-then" form is "if p, then q" or "if q, then p" by marking the appropriate column.
(10 points; 2 points each)

		Equivalent to…	
		If p, then q	If q, then p
a)	p if q	_________	_________
b)	p is necessary for q	_________	_________
c)	p is sufficient for q	_________	_________
d)	p whenever q	_________	_________
e)	p only if q	_________	_________

2) Consider the proposition r below

"I am going to Disneyland if I pass this class."

a) Write the <u>converse</u> of the proposition r in simple English.
(3 points)

b) Write the <u>contrapositive</u> of the proposition r in simple English.
(3 points)

3) Consider the following propositions:

p: Cartman will have to live with Kenny.
q: Cartman practices his singing.
r: Cartman takes singing lessons.
s: Cartman will win the Cheezy Poofs contest.

Completely rewrite the following proposition using symbolic logic operators and the propositional variables p, q, r, and s:

"If Cartman practices his singing or takes singing lessons, then he will win the Cheezy Poofs contest and will not have to live with Kenny."

(4 points)

4)

a) Write the truth table for the proposition $(\neg p \vee q) \wedge (\neg q \vee p)$. Show all work! (6 points)

b) Which one of the following is the proposition in a) logically equivalent to: $p \oplus q$, $p \rightarrow q$, or $p \leftrightarrow q$? (2 points)

5) Complete all of the columns in the truth table below to demonstrate that the proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. (8 points)

p	q	r	$(p \to q)$	$(q \to r)$	$[(p \to q) \wedge (q \to r)]$	$(p \to r)$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

6) Let the universe of discourse for x be $\{1,2,3\}$, and let $P(x)$ be a propositional function depending on x. Rewrite the proposition $\exists x\, P(x)$ as a logically equivalent expression containing just $P(1)$, $P(2)$, $P(3)$, and symbolic logical operators. (3 points)

7) Let $P(x,y)$ be a propositional function.
Let the universe of discourse for x be $\{x_1, x_2, x_3, x_4\}$.
Let the universe of discourse for y be $\{y_1, y_2, y_3, y_4\}$.
The grid below gives the truth value of $P(x_i,y_j)$ for $1 \le i \le 4$, $1 \le j \le 4$.

y_4	T	F	F	F
y_3	T	F	T	T
y_2	F	F	T	F
y_1	F	F	F	T
	x_1	x_2	x_3	x_4

For each proposition below, indicate whether it is True (T) or False (F).
(18 points; 3 points each)

a) $\forall x \forall y\ P(x,y)$ __________

b) $\exists x \exists y\ \neg P(x,y)$ __________

c) $\forall x \exists y\ P(x,y)$ __________

d) $\forall y \exists x\ P(x,y)$ __________

e) $\exists x \forall y\ \neg P(x,y)$ __________

f) $\exists y \forall x\ \neg P(x,y)$ __________

8) Let $P(x,y)$ be a propositional function, and let the universe of discourse for both x and y be the set of all real numbers. (6 points; 3 points each)

a) Yes or No:
If $\forall x \exists y\ P(x,y)$ is True, must $\exists y \forall x\ P(x,y)$ also be True?

b) Yes or No:
If $\exists y \forall x\ P(x,y)$ is True, must $\forall x \exists y\ P(x,y)$ also be True?

9) Rewrite the proposition $\neg \exists y \forall x\ Q(x,y)$ as a logically equivalent proposition containing only quantifiers, x, y, and $\neg Q(x,y)$. (3 points)

10) Let the universe of discourse for x be the set of <u>all integers</u>.
Let the universe of discourse for y be the set of <u>all positive integers</u>.
Let the universe of discourse for z be the set of <u>all positive integers</u>.
For each of the propositions below, indicate whether it is True (T) or False (F). Show work for f) and g). (34 points total)

a) $\exists y\,(y = 2y)$ __________ (3 points)

b) $\exists x\,(x = 2x)$ __________ (3 points)

c) $\forall x \exists y\,(x = y - 6)$ __________ (3 points)

d) $\forall y \exists x\,(x = y - 6)$ __________ (3 points)

e) $\exists y \exists z\,(yz = 0)$ __________ (3 points)

f) $\exists y \exists z\,(2y + 3z = 5 \wedge 4y + 2z = 7)$ __________
(Show work!) (5 points)

g) $\exists y \exists z\,(y - z = 1 \wedge 3y - 4z = 0)$ __________
(Show work!) (5 points)

h) $\forall x \forall y \exists z\,(xy = z)$ __________ (3 points)

i) $\exists x \exists y \forall z\,(xy = z)$ __________ (3 points)

j) $\forall y \forall z \exists x\,(x = yz)$ __________ (3 points)