Math 245   Name: _____

QUIZ 2
SECTIONS 1.4-1.7: BASIC STRUCTURES

1) Let T be the set $\{x \mid x$ is a prime number between 2 and 20, inclusive$\}$. Remember that a prime number is an integer greater than 1 that is divisible by no positive integer other than 1 and itself. (4 points)

    a) Write down the elements of T.

    b) What is the cardinality of T (denoted "$|T|$")?

2) Let S be the set $\{$Al, George, Pat, Ralph$\}$. (10 points; 2 points each)

    a) True or False: $\varnothing \in S$ _____

    b) True or False: $\varnothing \subseteq S$ _____

    c) Let $P(S)$ be the power set of S. What is $|P(S)|$?

    d) True or False: $\varnothing \in P(S)$ _____

    e) True or False: $\varnothing \subseteq P(S)$ _____

3) If the cardinality of the set A is m and the cardinality of the set B is n, where m and n are both nonnegative integers, what is the cardinality of $P(A \times B)$, the power set of $A \times B$, in terms of m and n ? (6 points)

4)   Let the universal set U be $\{x \mid x \in \mathbf{Z} \text{ and } 1 \le x \le 10\}$.
Let A_1 be the set $\{1, 2, 3, 4, 5\}$.
Let A_2 be the set $\{2, 4, 6, 8, 10\}$.
Let A_3 be the set $\{1, 2, 10\}$.

For each of a) through e), write down the elements of the set.
(20 points; 4 points each)

a) $A_1 - A_2$

b) $\overline{A_2}$

c) $A_3 \cup \varnothing$

d) $\bigcup_{i=1}^{3} A_i$

e) $\bigcap_{i=1}^{3} A_i$

5) Let A and B be subsets of some universal set U. (25 points total)

a) Use the basic set identities to prove

$$\overline{A \cap (B \cup \overline{A})} = \overline{A} \cup \overline{B}$$

(Hint: It's easier to use a Distributive Law first.)
For full credit, show all steps! (15 points)

b) Simplify $\overline{(A \cup A) \cap \overline{A}}$. Your answer will be one of the following:

$$U, A, \overline{A}, \text{ or } \varnothing$$

For full credit, show all steps! (10 points)

6)   (9 points total; 3 points each)

Let A be the set {1, 2, 3, 4}.
Let B be the set {a, b, c, d, e}.

Let the function $f : A \rightarrow B$ be defined as follows:

$$f(1) = c$$
$$f(2) = b$$
$$f(3) = a$$
$$f(4) = e$$

a) Yes or No: Is f a one-to-one function? _____

b) Yes or No: Is f an onto function? _____

c) Yes or No: Is f an invertible function? _____

7)   (14 points total)

Let S be the set $\{x \mid x \in \mathbf{Z} \text{ and } -2 \leq x \leq 4\}$.
Let the function $f : S \rightarrow \mathbf{Z}$ be defined as follows:

$$f(x) = 3x^6$$

a) What is the cardinality of the range of f?
   (Hint: No elaborate computation is required!)
   (5 points)

b) Yes or No: Is f a one-to-one function? _____ (3 points)

c) Yes or No: Is f an onto function? _____ (3 points)

d) Yes or No: Is f an invertible function? _____ (3 points)

8) Completely evaluate the following sum:

$$\sum_{j=1}^{3} \sum_{k=0}^{j-1} \left\lfloor \frac{j+k}{2} \right\rfloor$$

Show all work! (12 points)