## QUIZ 3
SECTIONS 2.3-2.5: NUMBER THEORY

Show all work where appropriate! Your proofs will be graded on quality, clarity, completeness, and correctness.

1) (24 points total; 6 points each)

   a) Find the prime factorization of 980.

   b) Find the prime factorization of 616.
c) Using a) and b), give the prime factorization of \( \text{lcm}(980,616) \) and then evaluate this \( \text{lcm} \).

d) Using a) and b), give the prime factorization of \( \text{gcd}(980,616) \) and then evaluate this \( \text{gcd} \).

2) I know that \( \text{gcd}(4743,867) = 51 \). Find \( \text{lcm}(4743,867) \).

   Hint: There is a shortcut! (5 points)

3) Prove: If \( n \) is a composite integer, then it has a nontrivial positive factor that is less than or equal to \( \sqrt{n} \). (10 points)
4) Find the highest integer \( n \) such that \( 3^n \mid 100! \); remember that 
\( n! = (1)(2)(3) \cdots (n) \) for positive integers \( n \). (10 points)

5) What is \( 1000 \mod 7 \)? (6 points)

6) Yes or No: Is \( 3709 \equiv 37 \pmod{51} \)? Justify your answer. (5 points)

7) Prove: If \( a \equiv b \pmod{m} \) and \( c \equiv d \pmod{m} \), then 
\( a + c \equiv b + d \pmod{m} \). Assume that \( a, b, c, d, \) and \( m \) are integers, 
with \( m \geq 2 \). (10 points)
8) The binary representation of a positive integer is 1001001. What is the decimal representation of this integer? (5 points)

9) The decimal representation of a positive integer is 182. What is the binary representation of this integer? (5 points)

10) Use the method shown in class to find two integers $s$ and $t$ such that $1 = 59s + 56t$. (10 points)
11) Use the method shown in class to find three positive integer solutions to the linear congruence $11x + 4 \equiv 13 \pmod{15}$.

- Hint: $1 = (15)(3) - (11)(4)$.
- A "brute-force" approach will not receive full credit!

(10 points)