Math 245 Name: ________________________

QUIZ 3

SECTIONS 2.3-2.5: NUMBER THEORY

Show all work where appropriate! Your proofs will be graded on quality, clarity, completeness, and correctness.

1) (24 points total; 6 points each)

a) Find the prime factorization of 980.

b) Find the prime factorization of 616.

c) Using a) and b), give the prime factorization of lcm(980,616) and then evaluate this lcm.

d) Using a) and b), give the prime factorization of gcd(980,616) and then evaluate this gcd.

2) I know that gcd(4743,867) = 51. Find lcm(4743,867).
Hint: There is a shortcut! (5 points)

3) Prove: If n is a composite integer, then it has a nontrivial positive factor that is less than or equal to $\sqrt{n}$. (10 points)

4) Find the highest integer n such that $3^n \mid 100!$; remember that $n! = (1)(2)(3) \cdots (n)$ for positive integers n. (10 points)

5) What is 1000 mod 7? (6 points)

6) Yes or No: Is $3709 \equiv 37 \pmod{51}$? Justify your answer. (5 points)

7) Prove: If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $a + c \equiv b + d \pmod{m}$. Assume that a, b, c, d, and m are integers, with $m \geq 2$. (10 points)

8) The binary representation of a positive integer is 1001001. What is the decimal representation of this integer? (5 points)

9) The decimal representation of a positive integer is 182. What is the binary representation of this integer? (5 points)

10) Use the method shown in class to find two integers s and t such that $1 = 59s + 56t$. (10 points)

11) Use the method shown in class to find three positive integer solutions to the linear congruence $11x + 4 \equiv 13 \pmod{15}$.

- Hint: $1 = (15)(3) - (11)(4)$.
- A "brute-force" approach will not receive full credit!

(10 points)