

QUIZ 4

SECTIONS 3.1-3.3: METHODS OF PROOF; INDUCTIVE REASONING

Show all work where appropriate! Your proofs will be graded on quality, clarity, completeness, and correctness.

- 1) Prove that $\sqrt{3}$ is irrational using methods suggested in Section 3.1.
(25 points)

2) Consider the conjecture: If the sum of four real numbers is less than 100, then at least one of the numbers is less than 25. (10 points total)

a) The conjecture essentially takes the form of an implication. Write (in English) the contrapositive of this implication. (5 points)

b) Is the conjecture a true or false statement? (5 points)

3) Use weak induction to prove

$$1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1$$

for all nonnegative integers n . (15 points)

4) Use weak induction to show that a total of n cents of postage (where n is any integer that is at least 20) can be obtained by using 4-cent stamps and/or 7-cent stamps. No other stamps are available. (25 points)

5) Use strong induction to prove $a_n \leq \left(\frac{4}{3}\right)^n$ for every nonnegative integer n , where the $\{a_n\}$ sequence is recursively defined as follows:

$$\begin{cases} a_0 = a_1 = a_2 = 1 \\ a_n = a_{n-2} + a_{n-3} \quad (\text{for } n \geq 3) \end{cases}$$

(25 points)