Math 245   Name: _____

QUIZ 5
SECTIONS 4.1-4.3: COUNTING

Show all work where appropriate! Write your name on all of your worksheets. For each problem, clearly indicate the problem number and/or letter, relevant work, and your answer.

<u>Note 1</u>: Unless otherwise specified, "or" means "inclusive or".

<u>Note 2</u>: You do <u>not</u> have to evaluate $C(n,r)$, or $\binom{n}{r}$. You may leave those expressions in your answers.

1) A bag contains 10 red balls, 10 blue balls, and 10 green balls. You will select balls at random from the bag without looking inside the bag.
(10 points; 5 points each)

   a) What is the minimum number of balls that you can pull out such that you can be sure that at least five of the selected balls will be of the same color?

   b) What is the minimum number of balls that you can pull out such that you can be sure that at least five of the selected balls will be green?

2) How many positive integers less than 5000 are divisible by 5 but not by 3? (10 points)

3) There are 26 English lowercase letters ("a" through "z"), including 5 vowels and 21 consonants. How many strings of six English lowercase letters are there that … (30 points; 10 points each)

   a) … begin with a vowel, end with a consonant, and have no letter that is repeated?

   b) … begin with a vowel or end with a vowel, provided that repetitions of letters are permitted?

   c) … have at least one consonant, provided that repetitions of letters are permitted?

4) Seven runners (including George and Dick) compete in a race. I know for a fact that:

- There are no ties.
- Exactly two runners finish after George but before Dick.

Given these restrictions, in how many different orders can the seven runners finish the race? (10 points)

5) An instructor tells me that she expects 13 of her 20 students to attend her class tomorrow, but she has no idea which 13 they will be. If she is correct, how many possible ways can her roll sheet for tomorrow be filled out, given that we don't know yet which 13 will show up? Assume that all "present" marks look alike and that all "absent" marks look alike.
(5 points).

6) Furthermore, the instructor from Problem 5 tells me that, of the 13 students who will attend tomorrow, at least 11 will be female. I know that, of the 20 students in her class, 12 are female and 8 are male. If she is correct, how many possible ways can her roll sheet for tomorrow be filled out, given what we know and assume now? Assume that all "present" marks look alike and that all "absent" marks look alike. (10 points)

7) How many bit strings of length 10 are there that have exactly 7 zeros ("0"s) and 3 ones ("1"s)? (5 points)

8) A bit string is also called a "binary string". A "ternary string" can be made up of "0"s, "1"s, and/or "2"s. How many ternary strings of length 8 are there that have exactly 3 zeros ("0"s)? (10 points)

9) There are four elected positions in Townsville: mayor, deputy mayor, treasurer, and dogcatcher. If there are 100 eligible adults in Townsville, how many ways can these positions be filled, assuming that no one can hold more than one position? (5 points)

10) A psychic tells me that Al Gore and G.W. Bush will each win 25 states in this year's presidential election. In addition, Gore will win the District of Columbia, which is not a state. Assuming he/she is right, how many possible "electoral maps" are there? (An "electoral map" is solely characterized by the distribution of the 50 states between Bush and Gore.) (5 points)