

QUIZ 6**SECTIONS 5.1, 5.2, 5.5:
RECURRENCE RELATIONS AND INCLUSION-EXCLUSION**

Show all work where appropriate! Write your name on all of your worksheets. For each problem, clearly indicate the problem number and/or letter, relevant work, and your answer.

Note: Unless otherwise specified, "or" means "inclusive or".

1) Find a recurrence relation for the number of bit strings of length n that contain four consecutive "1"s. (10 points)

2) A decimal string of length n consists of n digits. (A "digit" may be 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9). (25 points total)

a) Find a recurrence relation for the number of decimal strings of length n that do not contain three consecutive "0"s. (10 points)

b) Find the number of decimal strings of length 3 that do not contain three consecutive "0"s. (5 points)

c) Find the number of decimal strings of length 4 that do not contain three consecutive "0"s. (10 points)

3) Solve the recurrence relation $a_n = 7a_{n-1} - 6a_{n-2}$ ($n \geq 2$)

together with the initial conditions $\begin{cases} a_0 = 30 \\ a_1 = 20 \end{cases}$. (20 points)

4) Use an iterative approach to find the solution (as an explicit, closed-form formula) to the recurrence relation $a_n = a_{n-1} + 3n + 2$ ($n \geq 1$)

together with the initial condition $a_0 = 5$. (20 points)

5) Use the inclusion-exclusion principle to find the number of decimal strings of length 5 that:

- begin with three even digits, or
- begin with a "0", or
- end with three even digits.

Show all work! (25 points)