MATH 245: QUIZ 1 SOLUTIONS

1) Only c) and e) are equivalent to "If p, then q". The "Daddy/man" trick should help!

2) Rewrite the original statement as "If I pass this class, then I am going to Disneyland."
Converse: If I am going to Disneyland, then I pass this class.
Contrapositive: If I am not going to Disneyland, then I do not pass this class.

3) $(q \vee r) \to (s \wedge \neg p)$. Actually, the parentheses are not necessary if the order of operators mentioned in class is adopted.

4) a)

p	q	$\neg p$	q	$(\neg p \vee q)$	$\neg q$	p	$(\neg q \vee p)$	$(\neg p \vee q) \wedge (\neg q \vee p)$
T	T	F	T	T	F	T	T	T
T	F	F	F	F	T	T	T	F
F	T	T	T	T	F	F	F	F
F	F	T	F	T	T	F	T	T

b) $p \leftrightarrow q$, which is True exactly when both p and q have the same truth value.

5)

p	q	r	$(p \to q)$	$(q \to r)$	$[(p \to q) \wedge (q \to r)]$	$(p \to r)$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

The final column consists of all "T"s, so the given proposition is a tautology.

6) $P(1) \vee P(2) \vee P(3)$

7)

a) F: at least one box is an "F".
b) T: for the above reason. b) is the negation of a)!
c) F: x_2 can't "find" a y to make P True.
d) T: each y can "find" at least one x to make P True.
e) T: the x_2 column is all "F"s.
f) F: there is no row of "F"s.

8) a) No b) Yes
Check out the pictures in my lecture notes!

9) $\forall y \exists x \neg Q(x, y)$. Moving the "$\neg$" has the effect of "flipping" quantifiers.

10)

a) F: only y=0 would work, and 0 is outside the uod for y.

b) T: x=0 works, and 0 is in the uod for x.

c) F: whatever x is, only $y = x + 6$ will make the equation true, but if $x \leq -6$, only a nonpositive value for y will work. So, no "legal" y that will make the equation hold exists for $x \leq -6$.

d) T: whatever y is, $x = y - 6$ will make the equation true. y can only be a [positive] integer, so $y - 6$ can only be an integer and is thus a "legal" value for x.

e) F: the equation is true only when y=0 or z=0, but 0 falls outside the uods for both y and z.

f) F: the unique solution to the system is $\left(y = \frac{11}{8},\ z = \frac{3}{4}\right)$. This solution does not consist of only [positive] integers, so these are not "legal" values for y and z.

g) T: the unique solution to the system is ($y = 4$, $z = 3$). This solution consists of only positive integers, so these are "legal" values for y and z.

h) F: if $x \leq 0$, its product with any positive integer y will not be a positive integer.

i) F: there is no "magic" pair of x and y that will work for all possible values of z.

j) T: any two positive integers y and z will have an integer product.