MATH 245: QUIZ 5 SOLUTIONS

1)

a) An extreme scenario: It doesn't get any worse than this - you pick 4 reds, 4 blues, and 4 greens. I must get at least 5 balls of the same color by the 13^{th} ball.
Answer: **13**.

b) An extreme scenario: You pick the 10 reds, then the 10 blues; the next 5 balls must be green. You must get at least 5 greens by the 25^{th} ball.
Answer: **25**.

2)

I want to count the relevant integers that are divisible by 5, but I need to subtract the number of those integers that are also divisible by 15.

$$\left\lfloor \frac{4999}{5} \right\rfloor - \left\lfloor \frac{4999}{15} \right\rfloor = 999 - 333 = \mathbf{666}$$

3)

a) Start with the leftmost position, then do the rightmost position, and then do the intermediate positions from left to right:

$$\underset{V}{\underline{5}}\ \underline{24}\ \underline{23}\ \underline{22}\ \underline{21}\ \underset{C}{\underline{21}} \qquad \text{Product} = \mathbf{26{,}777{,}520}$$

b)

The number of strings that begin with a vowel:

$$\underset{V}{\underline{5}}\ \underline{26}\ \underline{26}\ \underline{26}\ \underline{26}\ \underline{26} \qquad \text{Product} = 5 \times 26^5$$

The number of strings that end with a vowel:

$$\underline{26}\ \underline{26}\ \underline{26}\ \underline{26}\ \underline{26}\ \underset{V}{\underline{5}} \qquad \text{Product} = 5 \times 26^5$$

<u>However</u>, we are double-counting the strings that begin <u>and</u> end with a vowel. By the inclusion-exclusion principle, I should subtract the number of these strings:

$$\underset{V}{\underline{5}}\ \underline{26}\ \underline{26}\ \underline{26}\ \underline{26}\ \underset{V}{\underline{5}} \qquad \text{Product} = 25 \times 26^4$$

Answer: $5 \times 26^5 + 5 \times 26^5 - 25 \times 26^4 = 10 \times 26^5 - 25 \times 26^4 =$ **107,389,360**.

c) We should take the total number of unrestricted 6-letter strings (26^6) and subtract the number of 6-letter strings with NO consonants (i.e., all vowels: 5^6).
Answer: $26^6 - 5^6 =$ **308,900,151**.

4)

Case 1: George finishes 1st, and Dick finishes 4th:

$$\underset{G}{\underline{1}}\ \underline{5}\ \underline{4}\ \underset{D}{\underline{1}}\ \underline{3}\ \underline{2}\ \underline{1} \qquad \text{Product} = 5! = 120$$

There are a total of 4 symmetric cases:

Place:	1	2	3	4	5	6	7
Case 1:	G			D			
Case 2:		G			D		
Case 3:			G			D	
Case 4:				G			D

Answer: $4 \times 5! =$ **480**.

5)

$$\binom{\mathbf{20}}{\mathbf{13}} = \mathbf{77{,}520}$$

6)

Case 1: 11 females and 2 males attend

(# ways to "pick" the females)(# ways to "pick" the males) = $\binom{12}{11}\binom{8}{2}$

Case 2: 12 females and 1 male attend

(# ways to "pick" the females)(# ways to "pick" the males) = $\binom{12}{12}\binom{8}{1}$

(There are only 12 females in the class, so we can't have 13 females attending.)

Answer: $\binom{12}{11}\binom{8}{2} + \binom{12}{12}\binom{8}{1} = 336 + 8 =$ **344**.

7)

$$\binom{\mathbf{10}}{\mathbf{7}} = \binom{\mathbf{10}}{\mathbf{3}} = \mathbf{120}.$$

8)

Case 1: The three "0"s are in the first three positions:

$$\frac{1}{\text{"0"}}\ \frac{1}{\text{"0"}}\ \frac{1}{\text{"0"}}\ \frac{2}{\text{"1" or "2"}}\ \frac{2}{\text{"1" or "2"}}\ \frac{2}{\text{"1" or "2"}}\ \frac{2}{\text{"1" or "2"}}\ \frac{2}{\text{"1" or "2"}}$$

Product = $2^5 = 32$

There are a total of $\binom{8}{3}$ symmetric cases, corresponding to the possible ways of "placing" the three "0"s.

Answer: $\mathbf{32\binom{8}{3} = 32(56) = 1792}$.

9)

$$\frac{100}{M}\ \frac{99}{DM}\ \frac{98}{T}\ \frac{97}{D}$$

Product = $\boldsymbol{P(100,4) = 94{,}109{,}400}$.

10)

$\binom{\mathbf{50}}{\mathbf{25}}$ = about 1.26×10^{14}