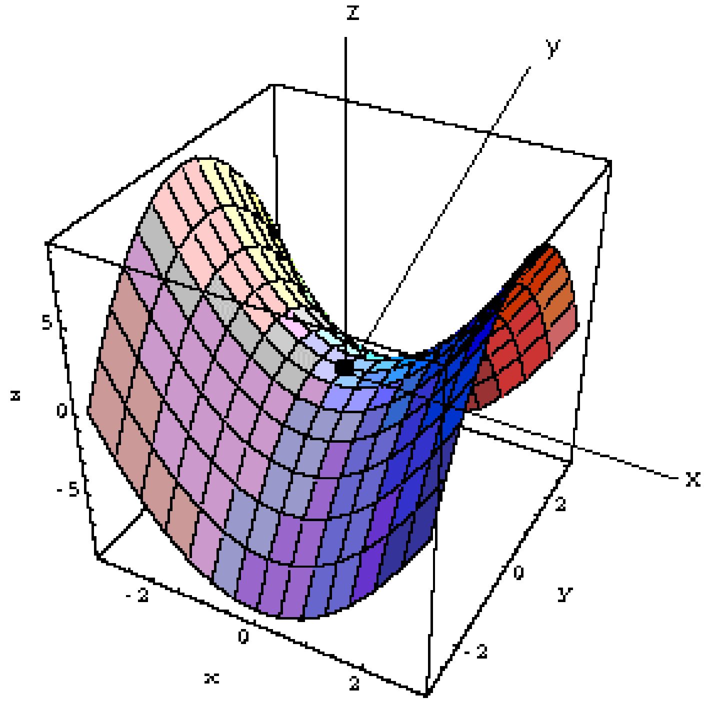
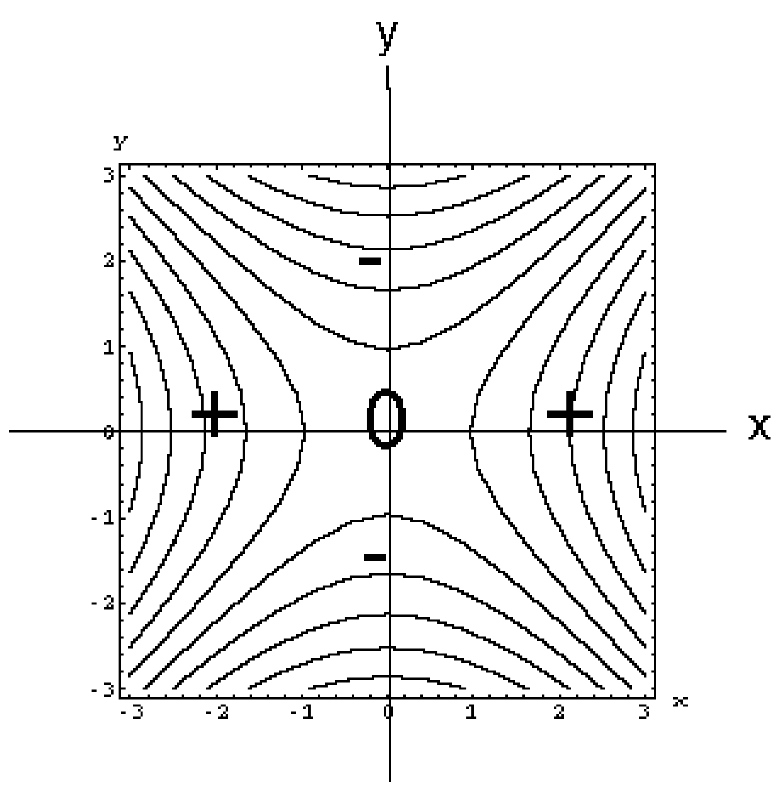
$f(x,y) = x^2 - y^2$

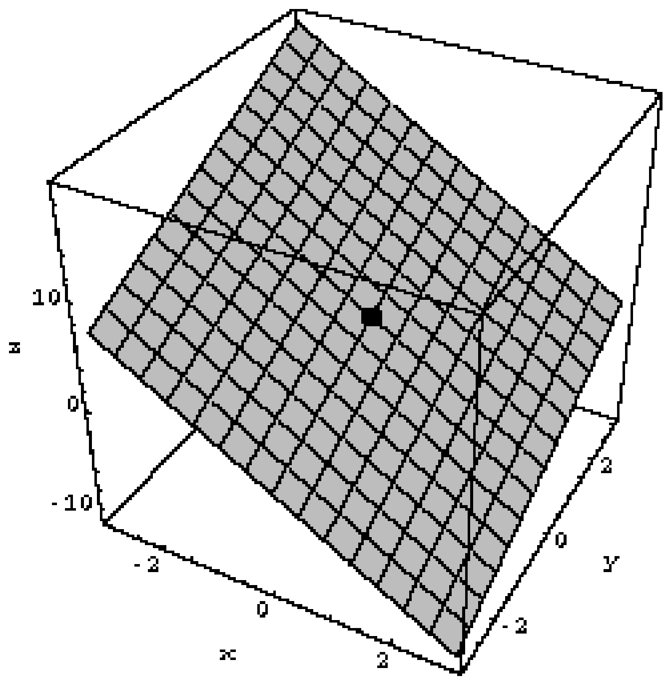
(A Hyperbolic Paraboloid; "Saddle")



Contour Plot



$f(x,y) = 4 - 3x + 2y$



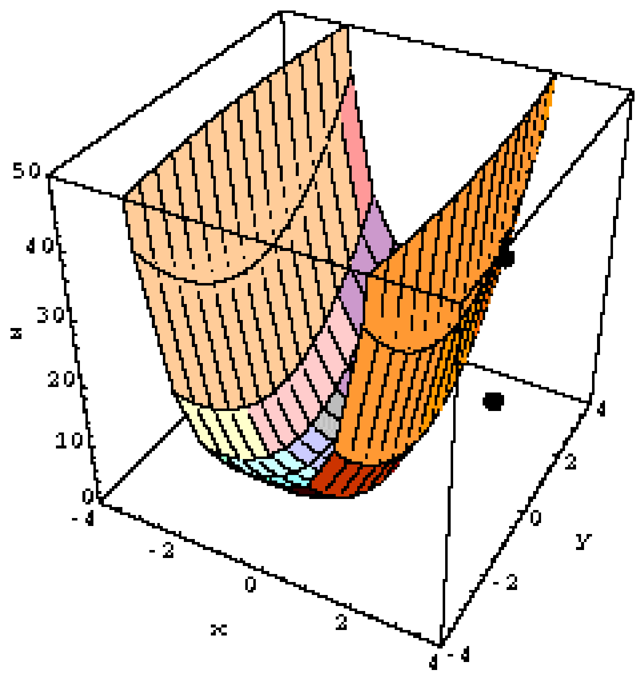
The x-slopes are –3 everywhere (i.e., at all points on the plane); the y-slopes are 2 everywhere.

If we fix any y-value (for example, $y = 0$, which corresponds to the x-axis), we get a cross-sectional line with a slope of –3 in the x-direction.

If we fix any x-value, (for example, $x = 0$, which corresponds to the y-axis) we get a cross-sectional line with a slope of +2 in the y-direction.

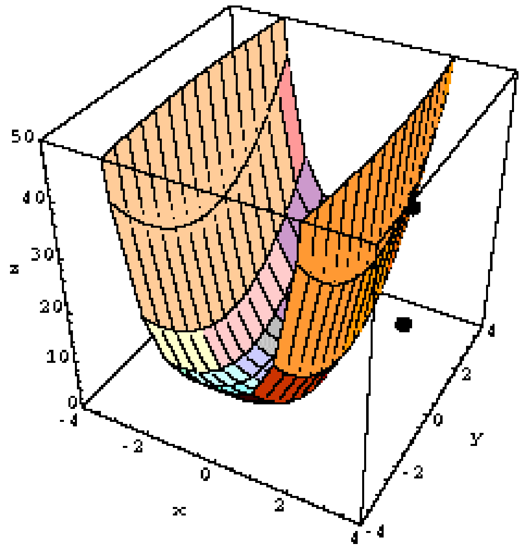
$f(x,y)$ or $z = x^4 + y^2$

I've plotted the points (2, 3, 0) and (2, 3, $f(2,3) = 25$), which lies on the surface.

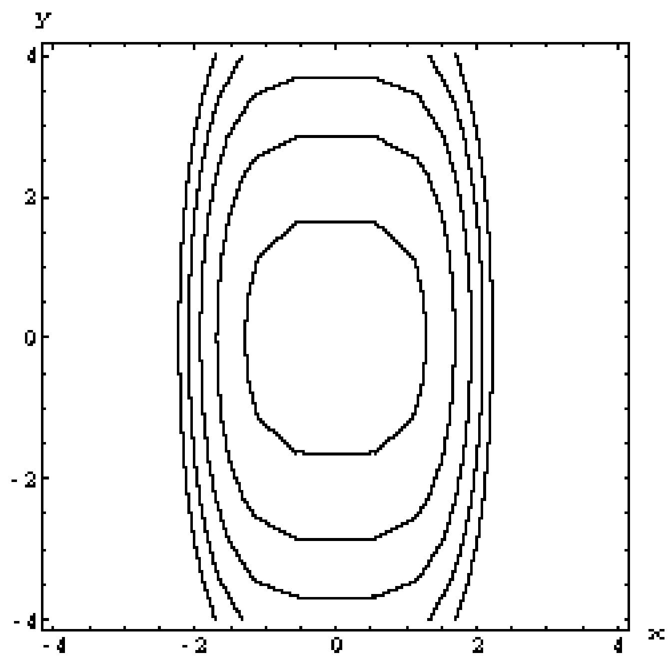


If we fix y to be some value k, we get $f(x,k) = x^4 + k^2 = x^4 +$ some number. Then, the corresponding cross-section is a steep quartic (fourth-degree) curve.

If we fix x to be some value k, we get $f(k,y) = k^4 + y^2 =$ some number $+ y^2$. Then, the corresponding cross-section is a not-as-steep quadratic (second-degree) curve.



Here's the corresponding contour diagram:



It turns out that $f_x(2,3) = 32$ and $f_y(2,3) = 6$; it makes sense that the former is larger, since $f(x,y)$ is a quartic in x but only a quadratic in y. The surface is much steeper in the x-direction than in the y-direction starting from (x=2,y=3). Given that the x- and y-scales are the same in our diagram, it is no wonder that the contours are closer in the x-direction than in the y-direction. In both directions, the contours are getting closer as we move away from (x=0, y=0), indicating that the surface becomes steeper and steeper as we move away from (x=0, y=0).

We have that grad $f(2,3) = [f_x(2,3)]\mathbf{i} + [f_y(2,3)]\mathbf{j} = 32\mathbf{i} + 6\mathbf{j}$. Note that this gradient pretty much points in the x-direction, with just a little tilt towards the y-direction. This makes sense, since [like a magic compass arrow] the gradient points in the direction where f increases most rapidly from the point you're at.