Math 252 Name: ________________________

FINAL

MATH 252 – FALL 2006 – KUNIYUKI
60 POINTS TOTAL (15 PROBLEMS; 4 POINTS EACH)

No books allowed. An appropriate sheet of notes and a scientific calculator are allowed.

1) What is the geometric definition of the dot product of two vectors **a** and **b** in V_n? Circle one:

   a) $\mathbf{a} \bullet \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos\theta$

   b) $\mathbf{a} \bullet \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \sin\theta$

   c) $\mathbf{a} \bullet \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \tan\theta$

2) Give symmetric equations for the line in *xyz*-space that passes through the point $(7, 4, -2)$ and that has direction vector $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

3) The graph of $3x^2 + 4y^2 - z = 0$ in *xyz*-space is … (circle one):

   a) A Cone

   b) An Ellipsoid

   c) An Elliptic Paraboloid

   d) A Hyperbolic Paraboloid

   e) A Hyperboloid of One Sheet

   f) A Hyperboloid of Two Sheets

4) A plane curve C is parameterized by $\mathbf{r}$, a smooth vector-valued function of t, from $t = a$ to $t = b$, where $a < b$. The curve does not overlap itself. Which of the following will give you the arc length of C? Circle one:

a) $\int_a^b \left\| \mathbf{r}(t) \right\| dt$ b) $\int_a^b \left\| \mathbf{r}'(t) \right\| dt$ c) $\int_a^b \left\| \mathbf{r}''(t) \right\| dt$

5) True or False: If $\mathbf{v}$ is a "nice" everywhere differentiable vector-valued function of t, $D_t\left[\mathbf{v}(t) \bullet \mathbf{v}(t)\right] = 2\left[\mathbf{v}'(t) \bullet \mathbf{v}(t)\right]$. Circle one:

True False

6) Two of the following are expressions for curvature that we have covered in class. Circle those two, and only those two.

a) $\left\| \dfrac{d\mathbf{r}}{ds} \right\|$ b) $\left\| \dfrac{d\mathbf{T}}{ds} \right\|$ c) $\left\| \dfrac{d\mathbf{N}}{ds} \right\|$

d) $\dfrac{\left\| \mathbf{r}(t) \times \mathbf{r}'(t) \right\|}{\left\| \mathbf{r}(t) \right\|^3}$ e) $\dfrac{\left\| \mathbf{r}'(t) \times \mathbf{r}''(t) \right\|}{\left\| \mathbf{r}'(t) \right\|^3}$

7) Let f be a "nice" differentiable function of x and y. Give the limit definition of $f_y(x, y)$.

8) Let S be the graph of $F(x, y, z) = 0$, where $\nabla F(x, y, z)$ is continuous.
If $\nabla F(2, -1, 3) = \langle 6, 2, 3 \rangle$, write an equation for the tangent plane to S at $(2, -1, 3)$.

9) Let f be a "nice" function of x and y with continuous second-order partial derivatives. Let $D = f_{xx} f_{yy} - (f_{xy})^2$. The point $(-1, -3)$ is a critical point of f where $D = -20$ and $f_{xx} = 5$. Which one of the following does f have at $(-1, -3)$? Circle one:

a) A local maximum

b) A local minimum

c) A saddle point

10) Express dV in cylindrical coordinates.

11) Express dV in spherical coordinates.

12) If $x = 2u - 3v$ and $y = 3u + 4v$, find the Jacobian $\dfrac{\partial(x, y)}{\partial(u, v)}$.

Hints: The Jacobian is computed using a determinant. Your answer will be a number.

13) Consider the work applied by a force $\mathbf{F}(x,y)=\langle M(x,y), N(x,y)\rangle$ on a particle traveling along a curve *C*. Three of the following formulas are work formulas that we have covered in class. Circle those three.

a) $\int_C M\,dx + N\,dy$ b) $\int_C \mathbf{F} \bullet \mathbf{r}'(t)\,dt$ c) $\int_C \mathbf{F} \bullet \mathbf{N}\,ds$

d) $\int_C \mathbf{F} \bullet \mathbf{T}\,ds$ e) $\int_C \mathbf{F} \bullet d\mathbf{T}$

14) Which two of the following each guarantees that a vector field **F** is conservative throughout $\mathbf{R}^3$? Circle two:

a) If C is any circle in $\mathbf{R}^3$ that encloses the origin, $\oint_C \mathbf{F} \bullet d\mathbf{r} = 0$.

b) $\mathbf{curl\,F} = \mathbf{0}$ throughout $\mathbf{R}^3$.

c) $\mathbf{F} = \nabla f$ for some scalar function f throughout $\mathbf{R}^3$.

15) Assume that the hypotheses of Green's Theorem (as stated in my 18.4 Notes) are satisfied. In particular, C is a piecewise smooth simple closed curve in the *xy*-plane that is the boundary of R, which consists of C and its interior. D is an open region containing R. $\mathbf{F}(x,y)=\langle M(x,y), N(x,y)\rangle$, where *M* and *N* are "nice" in *D*. Fill in the blank:

According to Green's Theorem,

$$\oint_C \mathbf{F} \bullet d\mathbf{r}, \text{ or } \oint_C M\,dx + N\,dy = \underline{\hspace{6cm}}.$$