Math 252 Name: ___________________________

FINAL

MATH 252 – FALL 2008 – KUNIYUKI
60 POINTS TOTAL (15 PROBLEMS; 4 POINTS EACH)

No books allowed. An appropriate sheet of notes and a scientific calculator are allowed.

1) If **a** and **b** are vectors in V_2 such that $\mathbf{a} \bullet \mathbf{b} < 0$, which of the following is true? Box in the best answer:

   a) The angle between **a** and **b** is in $\left[0^\circ, 90^\circ\right)$.

   b) The angle between **a** and **b** is in $\left(90^\circ, 180^\circ\right]$.

   c) $\mathbf{a} \bullet \mathbf{b}$ tells us nothing about the angle between **a** and **b**.

2) Write an equation for the plane in *xyz*-space that is parallel to the plane with equation $3x - 2y + 7z = 10$ and that contains the point $(4, -1, 8)$.

3) Consider the graph of $4x^2 - 5y^2 - 6z^2 = 3$ in *xyz*-space. The trace of the graph in the plane $x = 10$ is … (Box in one):

   a) An Ellipse

   b) A Hyperbola

   c) A Parabola

4) If **r** is a vector-valued position function of t that is smooth and twice differentiable for all real t, which one of the following will be true for all real t? Box in one:

a) $\mathbf{r}(t) \perp \mathbf{T}(t)$

b) $\mathbf{r}'(t) \perp \mathbf{T}(t)$

c) $\mathbf{r}'(t) \parallel \mathbf{T}(t)$

5) If the curvature at a point on a curve (in the real plane) is 10, what is the radius of curvature at that point?

6) Fill in the blank: The level surface of $f(x, y, z) = 4x^2 + 9y^2 - z^2,\ k = 4$ is _____. (Pick a letter from below.)

A. A Sphere or Ellipsoid
B. A Hyperboloid of One Sheet
C. A Hyperboloid of Two Sheets
D. A Cone
E. A Circular or Elliptic Paraboloid
F. A Hyperbolic Paraboloid
G. A Right Circular or Elliptic Cylinder
H. A Plane
I. A Line (a "degenerate" surface)
J. A Point (a "degenerate" surface)
K. NONE (no surface)

7) Assume that f is a differentiable function of x, y, and z. Give the limit definition of $f_z(x, y, z)$.

8) Let S be the graph of $F(x, y, z) = 0$, where $\nabla F(x, y, z)$ is continuous. Assuming that $\nabla F(-2, 4, 1) = \langle 8, 1, -3 \rangle$, write parametric equations for the normal line to S at $(-2, 4, 1)$.

9) Assume that f is a function of x and y with continuous second-order partial derivatives. Let $D = f_{xx} f_{yy} - \left(f_{xy} \right)^2$. The point $(2, -3)$ is a critical point of f where $D = 40$ and $f_{xx} = 2$. Which one of the following does f have at $(2, -3)$? Box in one:

a) A local maximum

b) A local minimum

c) A saddle point

10) The graph of $z = r$ $(r \geq 0)$ in *xyz*-space is part of … (Box in one):

A cone A circular cylinder A plane A sphere

11) Express dV in spherical coordinates.

12) If $x = 4u + 3v$ and $y = 2u - v$, find the Jacobian $\dfrac{\partial(x, y)}{\partial(u, v)}$.

13) Consider a vector field **F** that is "nice" throughout $\mathbf{R}^3$; here, "nice" means that the partial derivatives exist for the **i**-, **j**-, and **k**-components of **F** throughout $\mathbf{R}^3$. Assume that, at the origin, $\text{div}\,\mathbf{F} > 0$. Which of the following is at the origin? (Optional Hint: Try to find a "nice" **F** for which $\text{div}\,\mathbf{F} > 0$ throughout $\mathbf{R}^3$, and visualize it.) Box in one:

A sink   A source

14) Consider the work applied by a force $\mathbf{F}(x,y) = \langle M(x,y), N(x,y) \rangle$ on a particle traveling along a curve *C*. Two of the following formulas are work formulas that we have covered in class. Box in those two.

a) $\int_C \mathbf{F} \bullet \mathbf{r}(t)\, dt$ b) $\int_C \mathbf{F} \bullet \mathbf{T}'(t)\, dt$ c) $\int_C \mathbf{F} \bullet \mathbf{T}\, ds$

d) $\int_C M\, dx + N\, dy$ e) $\int_C \mathbf{F} \bullet \mathbf{N}\, ds$

15) If a vector field **F** is continuous in $\mathbf{R}^3$, under what conditions will we be guaranteed that the work integral $\int_C \mathbf{F} \bullet d\mathbf{r} = 0$? Box in one of a), b), or c): (Assume that a circular path corresponds to exactly one revolution.)

a) **F** is conservative in $\mathbf{R}^3$, and *C* is piecewise smooth.

b) $\text{div}\,\mathbf{F} = 0$ throughout $\mathbf{R}^3$, and *C* is a circle in $\mathbf{R}^3$.

c) $\mathbf{F} = \nabla f$ throughout $\mathbf{R}^3$, where $f(x,y,z) = xyz$, and *C* is a circle in $\mathbf{R}^3$.