JUMBLING TSPs

How is $(\mathbf{a}\times\mathbf{b})\bullet\mathbf{c}$ related to $(\mathbf{a}\times\mathbf{c})\bullet\mathbf{b}$?

Determinant approach:

$$(\mathbf{a}\times\mathbf{b})\bullet\mathbf{c}=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$(\mathbf{a}\times\mathbf{c})\bullet\mathbf{b}=\begin{vmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

The determinant forms differ only by a single switch of two rows, so they differ only by a sign.

$$\boxed{(\mathbf{a}\times\mathbf{b})\bullet\mathbf{c}=-\left[(\mathbf{a}\times\mathbf{c})\bullet\mathbf{b}\right]}$$

Geometric approach:

Both $\left|(\mathbf{a}\times\mathbf{b})\bullet\mathbf{c}\right|$ and $\left|(\mathbf{a}\times\mathbf{c})\bullet\mathbf{b}\right|$ represent the volume of the parallelepiped determined by the position vectors for **a**, **b**, and **c**. This is consistent with the box above.

How is $(\mathbf{a}\times\mathbf{b})\bullet\mathbf{c}$ related to $(\mathbf{b}\times\mathbf{c})\bullet\mathbf{a}$?

Determinant approach:

$$(\mathbf{a}\times\mathbf{b})\bullet\mathbf{c}=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$(\mathbf{b}\times\mathbf{c})\bullet\mathbf{a}=\begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

The determinant forms differ by <u>two</u> switches of pairs of rows, so there is a "double negative" effect, and the determinants are equal.

$$\boxed{(\mathbf{a}\times\mathbf{b})\bullet\mathbf{c}=(\mathbf{b}\times\mathbf{c})\bullet\mathbf{a}}$$

Geometric approach:

(Similar to the first example)

Remember that dot products are commutative, and cross products are anticommutative, so it may be easy to relate some jumbles. For example,

$$\begin{aligned}(\mathbf{a}\times\mathbf{b})\bullet\mathbf{c}&=\mathbf{c}\bullet(\mathbf{a}\times\mathbf{b})\\ &=-\left[\mathbf{c}\bullet(\mathbf{b}\times\mathbf{a})\right]\end{aligned}$$

Basically, if you perform a TSP jumble of three vectors in V_3 that makes sense (you only take dot products of two vectors of the same length, and you only take cross products of two vectors in V_3), you either get the original TSP value or its opposite. This is consistent with the geometric interpretation of the absolute value of a TSP as the volume of a parallelepiped determined by the position vectors for the three constituent vectors.