

Find $\frac{\partial}{\partial y}$ of the following
(remember, x acts like a
constant):

1) $e^{xx} y^2$

2) $y^2 e^{x^2 y^3}$

3) $\frac{y^2}{\ln y}$

4) $\cos^3(xy^2)$

5) $\tan^{-1}(xy^2)$

6) $\sin^3 y$

7) $\sin(y^3)$

8) $\ln\sqrt{x^2 + y^2}$

Answers

(not necessarily simplified):

$$1) \frac{\partial}{\partial y} \left(e^{xx} y^2 \right) = e^{xx} (2y)$$

e^{xx} acts like a constant multiplier for y^2

$$\begin{aligned} 2) \frac{\partial}{\partial y} \left(y^2 e^{x^2 y^3} \right) &= (2y) e^{x^2 y^3} + y^2 \left(e^{x^2 y^3} \bullet \frac{\partial}{\partial y} \left(x^2 y^3 \right) \right) \\ &= (2y) e^{x^2 y^3} + y^2 \left(e^{x^2 y^3} \bullet x^2 (3y^2) \right) \end{aligned}$$

Product rule and chain rule

$$3) \frac{\partial}{\partial y} \left(\frac{y^2}{\ln y} \right) = \frac{(\ln y)(2y) - (y^2) \left(\frac{1}{y} \right)}{(\ln y)^2}$$

Quotient rule: $\frac{\text{Lo} \bullet \text{D(Hi)} - \text{Hi} \bullet \text{D(Lo)}}{\text{Square of below}}$

As in 2), write $\frac{\partial}{\partial y}$ if you need the chain rule.

$$4) \frac{\partial}{\partial y} \left(\cos^3(xy^2) \right) = \frac{\partial}{\partial y} \left[\left(\cos(xy^2) \right)^3 \right] \quad (\text{Clearer notation})$$

$$= 3 \left(\cos(xy^2) \right)^2 \bullet \frac{\partial}{\partial y} \left(\cos(xy^2) \right)$$

$$= 3 \left(\cos(xy^2) \right)^2 \bullet \left(-\sin(xy^2) \bullet \frac{\partial}{\partial y} (xy^2) \right)$$

$$= 3 \left(\cos(xy^2) \right)^2 \bullet \left(-\sin(xy^2) \bullet x(2y) \right)$$

Trig powers, power rule, chain rule (twice!)

$$\begin{aligned}
 5) \quad \frac{\partial}{\partial y} \left(\tan^{-1}(xy^2) \right) &= \frac{1}{1+(xy^2)^2} \bullet \frac{\partial}{\partial y} (xy^2) \\
 &= \frac{1}{1+(xy^2)^2} \bullet x(2y)
 \end{aligned}$$

$$\text{Rule: } \frac{\partial}{\partial y} \tan^{-1}(\text{blah}) = \frac{1}{1+\text{blah}^2} \bullet \frac{\partial}{\partial y} (\text{blah})$$

$$\begin{aligned}
 6) \quad \frac{\partial}{\partial y} (\sin^3 y) &= (\sin y)^3 \quad (\text{Clearer notation}) \\
 &= 3(\sin y)^2 \bullet \frac{\partial}{\partial y} (\sin y) \\
 &= 3(\sin y)^2 \bullet \cos y
 \end{aligned}$$

$$\begin{aligned}
 7) \quad \frac{\partial}{\partial y} \left(\sin(y^3) \right) &= \cos(y^3) \bullet \frac{\partial}{\partial y} (y^3) \\
 &= \cos(y^3) \bullet 3y^2
 \end{aligned}$$

As opposed to 6), we don't need the power rule here until the end.

$$\begin{aligned}
8) \quad \frac{\partial}{\partial y} \left(\ln \sqrt{x^2 + y^2} \right) &= \frac{1}{\sqrt{x^2 + y^2}} \bullet \frac{\partial}{\partial y} \left(\sqrt{x^2 + y^2} \right) \\
&= \frac{1}{\sqrt{x^2 + y^2}} \bullet \frac{\partial}{\partial y} \left[\left(x^2 + y^2 \right)^{1/2} \right] \\
&= \frac{1}{\sqrt{x^2 + y^2}} \bullet \frac{1}{2} \left(x^2 + y^2 \right)^{-1/2} \bullet \frac{\partial}{\partial y} \left(x^2 + y^2 \right) \\
&= \frac{1}{\sqrt{x^2 + y^2}} \bullet \frac{1}{2} \left(x^2 + y^2 \right)^{-1/2} \bullet 2y
\end{aligned}$$

$$\text{Rule: } \frac{\partial}{\partial y} \ln(\text{blah}) = \frac{1}{\text{blah}} \bullet \frac{\partial}{\partial y} (\text{blah})$$

Roots as powers, power rule, chain rule