Math 252 Name: ________________________

# QUIZ 1 (CHAPTER 14)

**MATH 252 – FALL 2007 – KUNIYUKI**
**SCORED OUT OF 125 POINTS ⇒ MULTIPLIED BY 0.84 ⇒ 105% POSSIBLE**

**Show all work, simplify as appropriate, and use "good form and procedure" (as in class).**
**Box in your final answers!**
**No notes or books allowed. A scientific calculator is allowed.**

Clearly mark vectors, as we have done in class. I will use boldface, but you don't!
When describing vectors, you may use either $\langle\ \rangle$ or "$\mathbf{i}-\mathbf{j}-\mathbf{k}$" notation.
Assume we are in our usual 2- and 3-dimensional Cartesian coordinate systems.
Give exact answers, unless otherwise specified.

Check one:

Can you easily print files from the class web site?

☐ Yes. I do **not** need copies of exam solutions made for me.

☐ No. Please provide me with copies of exam solutions.

**USE THE LAST PAGE IF YOU NEED MORE SPACE!**

1) Assume that $a_1$, $a_2$, $p$, and $q$ are real numbers.
Prove that, if $\mathbf{a} = \langle a_1, a_2 \rangle$, then $(p+q)\mathbf{a} = p\mathbf{a} + q\mathbf{a}$. Show <u>all</u> steps! (10 points)

2) Write an inequality in $x$, $y$, and/or $z$ whose graph in our usual three-dimensional $xyz$-coordinate system consists of the sphere of radius 4 centered at the origin and all points inside that sphere. (4 points)

3) Find all real values of $c$ such that the vectors $c\mathbf{i}+10\mathbf{j}+c\mathbf{k}$ and $c\mathbf{i}-2\mathbf{j}-\mathbf{k}$ are orthogonal. (8 points)

4) Assume that **a** and **b** are vectors in $V_n$, where $n$ is some natural number. Using entirely mathematical notation (i.e., don't use words) …
(8 points; 4 points each)

   a) Write the Cauchy-Schwarz Inequality.

   b) Write the Triangle Inequality.

5) Let **a**, **b**, and **c** be vectors in $V_3$. (4 points total; 2 points each)

a) $(\mathbf{a} \bullet \mathbf{b})\mathbf{c}$ is … (Box in one:)

a scalar    a vector    neither, or undefined

b) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ is … (Box in one:)

a scalar    a vector    neither, or undefined

6) Assume that **a**, **b**, and **c** are three nonzero vectors in $V_3$ such that $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$. Which of the following must be true? Box in one: (3 points)

i) The vector **a** and the vector $\mathbf{b} - \mathbf{c}$ are parallel.

ii) The vector **a** and the vector $\mathbf{b} - \mathbf{c}$ are perpendicular (or orthogonal).

iii) $\mathbf{b} = \mathbf{c}$.

7) The line *l* passes through the points $P(-7, 2, 0)$ and $Q(4, -1, 5)$. (10 points total)

a) Find parametric equations for *l*.

b) Find symmetric equations for *l*.

8) Consider the following two lines:

$$l_1 : \begin{cases} x = 3 + t \\ y = 1 - 2t \\ z = -5 + 3t \end{cases} \quad \text{and} \quad l_2 : \begin{cases} x = -2 - 3u \\ y = 3 + 4u \\ z = 8 - 2u \end{cases} \qquad (t, u \in \mathbf{R})$$

(35 points total)

a) Find the point of intersection between the two lines.

b) Find either one of the two supplementary angles between the given lines.
Reminder:

$$l_1 : \begin{cases} x = 3 + t \\ y = 1 - 2t \\ z = -5 + 3t \end{cases} \quad \text{and} \quad l_2 : \begin{cases} x = -2 - 3u \\ y = 3 + 4u \\ z = 8 - 2u \end{cases} \qquad (t, u \in \mathbf{R})$$

Round off your answer to the nearest tenth of a degree.

c) Find an equation (in $x$, $y$, and $z$) of the plane that contains the two given lines. Reminder:

$$l_1 : \begin{cases} x = 3 + t \\ y = 1 - 2t \\ z = -5 + 3t \end{cases} \quad \text{and} \quad l_2 : \begin{cases} x = -2 - 3u \\ y = 3 + 4u \\ z = 8 - 2u \end{cases} \qquad (t, u \in \mathbf{R})$$

9) Consider the point $P(7, 2, -1)$ and the plane $4x - 3y + 2z + 60 = 0$.
Distance is measured in meters. (16 points total)

a) At what point does the given plane intersect the $x$-axis?
(We will call this point $Q$.)

b) Find a normal vector for the given plane. (We will call this vector **n**.)

c) If we let the vector $\mathbf{p} = \overrightarrow{QP}$, then the distance between the given point $P$ and the given plane equals: $\left|\text{comp}_{\mathbf{n}}\mathbf{p}\right|$. Use the component formula to find $\left|\text{comp}_{\mathbf{n}}\mathbf{p}\right|$. Round it off to the nearest tenth of a meter.

10) Matching. (12 points total)

Fill in each blank below with one of the following:

A. An Ellipsoid
B. A Hyperboloid of One Sheet
C. A Hyperboloid of Two Sheets
D. A Cone
E. An Elliptic Paraboloid
F. A Hyperbolic Paraboloid

I. The graph of $\frac{1}{2}x^2 - 3y^2 - z^2 = 5$ is _____.

II. The graph of $x^2 + 7y^2 - z = 0$ is _____.

III. The graph of $4x^2 - 9y^2 + z^2 = 0$ is _____.

IV. The graph of $4x^2 - y^2 + 11z^2 = 7$ is _____.

11) Consider the graph of $4x^2 - y^2 + 11z^2 = 7$. This was in Problem 10, part IV. Assume that $k$ takes the place of real constants. (12 points total)

a) The axis of the graph is the … (Box in one:)

$x$-axis $y$-axis $z$-axis

b) The conic traces of the graph in the planes $x = k$ are … (Box in one:)

Ellipses Hyperbolas Parabolas

c) The conic traces of the graph in the planes $y = k$ are … (Box in one:)

Ellipses Hyperbolas Parabolas

d) The conic traces of the graph in the planes $z = k$ are … (Box in one:)

Ellipses Hyperbolas Parabolas

12) Find an equation (in $x$, $y$, and $z$) of the surface obtained by revolving the graph of $4y^2 + 25z^2 = 1$ (in the $yz$-plane) about the $z$-axis. (3 points)