Math 252 Name: ________________________

QUIZ 1 (CHAPTER 14)

MATH 252 – FALL 2008 – KUNIYUKI
SCORED OUT OF 125 POINTS ⇒ MULTIPLIED BY 0.84 ⇒ 105% POSSIBLE

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

Clearly mark vectors, as we have done in class. I will use boldface, but you don't!
When describing vectors, you may use either $\langle \ \rangle$ or "$\mathbf{i} - \mathbf{j} - \mathbf{k}$" notation.
Assume we are in our usual 2- and 3-dimensional Cartesian coordinate systems.
Give exact answers, unless otherwise specified.

Check one:

Can you easily print files from the class web site?

☐ Yes. I do **not** need copies of exam solutions made for me.

☐ No. Please provide me with copies of exam solutions.

USE THE BOTTOM OF THE LAST PAGE IF YOU NEED MORE SPACE!

1) Consider the points $P(1, 3)$ and $Q(5, 8)$. Find the vector in V_2 that has the same direction as the vector [corresponding to] $\overrightarrow{PQ}$ and has length 100. (8 points)

2) Write an equation (in x, y, and z) of the sphere with center $(0, 4, -2)$ that is tangent to the xy-plane in our usual three-dimensional xyz-coordinate system. (4 points)

3) Assume that **a**, **b**, and **c** are nonzero vectors in V_3.

   Prove: $\text{comp}_{\mathbf{c}}(\mathbf{a}+\mathbf{b}) = \text{comp}_{\mathbf{c}}\mathbf{a} + \text{comp}_{\mathbf{c}}\mathbf{b}$.

   You may use the comp formula and the basic dot product properties listed in the book without proof. (Writing $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and so forth will not help here.)

   (6 points)

4) Write the Cauchy-Schwarz Inequality. Let **a** and **b** be vectors in V_n, where n is some natural number. (4 points)

5) You do <u>not</u> have to show work for this problem. There are many possible answers for part a) and for part b). (6 points total; 3 points each)

a) Assuming $\mathbf{v} = \langle 2, 3 \rangle$ in V_2, find a nonzero vector **w** in V_2 that is orthogonal to **v**.

b) Assuming $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ in V_3, find a nonzero vector **b** in V_3 such that $\mathbf{a} \times \mathbf{b} = \mathbf{0}$.

6) Let **a**, **b**, and **c** be vectors in V_3. (4 points total; 2 points each)

a) $(\mathbf{a} \times \mathbf{b}) \bullet \mathbf{c}$ is … (Box in one:)

a scalar   a vector   neither, or undefined

b) $\mathbf{a} \times (\mathbf{b} \bullet \mathbf{c})$ is … (Box in one:)

a scalar   a vector   neither, or undefined

7) The line *l* passes through the points $P(4, 1, -3)$ and $Q(6, -1, -8)$. (10 points total)

a) Find parametric equations for *l*.

b) Find symmetric equations for *l*.

8) Consider the following two lines:

$$l_1 : \begin{cases} x = 13 + 4t \\ y = 5 + 2t \\ z = -2 + 3t \end{cases} \quad \text{and} \quad l_2 : \begin{cases} x = -5 - 3u \\ y = -9 + u \\ z = -8 - 6u \end{cases} \qquad \left(t, u \in \mathbf{R}\right)$$

(31 points total)

a) Find the point of intersection between the two lines.

b) Find either one of the two supplementary angles between the given lines.
Reminder:

$$l_1 : \begin{cases} x = 13 + 4t \\ y = 5 + 2t \\ z = -2 + 3t \end{cases} \quad \text{and} \quad l_2 : \begin{cases} x = -5 - 3u \\ y = -9 + u \\ z = -8 - 6u \end{cases} \qquad (t, u \in \mathbf{R})$$

Round off your answer to the nearest tenth of a degree.

c) At what point does the line l_2 intersect the *yz*-plane? Reminder:

$$l_2 : \begin{cases} x = -5 - 3u \\ y = -9 + u \\ z = -8 - 6u \end{cases} \quad \left(u \in \mathbf{R}\right)$$

9) Consider the points $P(0, 4, 2)$, $Q(-1, 7, -3)$, and $R(2, 1, -1)$.
Distance is measured in meters. (20 points total)

a) Find an equation (in x, y, and z) of the plane containing the three points P, Q, and R.

b) Find the area of triangle *PQR*, the triangle that has the three given points (*P*, *Q*, and *R*) as its vertices. You may refer to your work in part a). Round off your answer to the nearest tenth of a square meter.

10) Find the distance between the point $(5, 2, -4)$ and the plane $3x - y + 2z + 7 = 0$. Distance is measured in meters. Round off your answer to the nearest tenth of a meter. (8 points)

11) Matching. (12 points total)

Fill in each blank below with one of the following letters (A-F):

A. An Ellipsoid
B. A Hyperboloid of One Sheet
C. A Hyperboloid of Two Sheets
D. A Cone
E. An Elliptic Paraboloid
F. A Hyperbolic Paraboloid

I. The graph of $3x^2 - 5y + 4z^2 = 0$ is _____.

II. The graph of $4z^2 - x^2 - 5y^2 = 2$ is _____.

III. The graph of $x^2 = y^2 + \frac{2}{5}z^2$ is _____.

IV. The graph of $y = 2x^2 - 3z^2$ is _____.

12) Consider the graph of $y = 2x^2 - 3z^2$. This was in Problem 11, part IV. Assume that k takes the place of real constants. (9 points total)

a) The conic traces of the graph in the planes $x = k$ are … (Box in one:)

Ellipses Hyperbolas Parabolas

b) The conic traces of the graph in the planes $y = k$ are … (Box in one:)

Ellipses Hyperbolas Parabolas

c) The conic traces of the graph in the planes $z = k$ are … (Box in one:)

Ellipses Hyperbolas Parabolas

13) Find an equation (in *x*, *y*, and *z*) of the surface obtained by revolving the graph of $16x^2 - 9z^2 = 1$ (in the *xz*-plane) about the *z*-axis. (3 points)