Math 252 Name: ________________________

QUIZ 2 (CHAPTER 15, 16.1, 16.2)

MATH 252 – FALL 2007 – KUNIYUKI
SCORED OUT OF 125 POINTS ⇒ MULTIPLIED BY 0.84 ⇒ 105% POSSIBLE

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

Clearly mark vectors, as we have done in class. I will use boldface, but you don't! When describing vectors or vector-valued functions, you may use either $\langle \ \rangle$ or "$\mathbf{i}-\mathbf{j}-\mathbf{k}$" notation.

USE THE BACK OF THIS TEST IF YOU NEED MORE SPACE!!

1) Find the length of the curve parameterized by:

$$x = t^2, \quad y = \frac{\sqrt{5}}{2}t^2, \quad z = 2t, \quad 0 \le t \le 2 .$$

Major Hint (which you may use without proof):
According to the Table of Integrals, if a is a positive real constant,

$$\int \sqrt{a^2 + u^2}\, du = \frac{u}{2}\sqrt{a^2 + u^2} + \frac{a^2}{2}\ln\left| u + \sqrt{a^2 + u^2} \right| + C$$

Leave your answer as a simplified exact answer; do not approximate it using a calculator. You do not have to apply log properties at the end. Distance is measured in meters. Show all work! (20 points)

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2) A curve C is parameterized by the vector-valued function (VVF) given by $\mathbf{r}(t)=\left\langle e^{2t}, 3t+1, t^2\right\rangle$. Find a tangent vector to C at the point $\left(e^{10}, 16, 25\right)$.
(10 points)

3) Complete the Product Rule for differentiating the dot product of two differentiable vector-valued functions (VVFs) **u** and **v**:

$$D_t\left[\mathbf{u}(t)\bullet\mathbf{v}(t)\right]= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

(3 points)

4) The velocity of a moving particle is given by $\mathbf{v}(t) = \langle 7e^t,\ 4\cos t,\ 3t^2 - 2 \rangle$.

Find the position vector-valued function (VVF rule) $\mathbf{r}(t)$ if $\mathbf{r}(0) = \langle 1, 4, -3 \rangle$.

(10 points)

5) Find the unit tangent VVF (rule) $\mathbf{T}(t)$ and the principal unit normal VVF (rule) $\mathbf{N}(t)$ for the curve C determined by $\mathbf{r}(t)=\langle -6t, 2t^3\rangle$, where $t>0$. Show all work and simplify completely, as we have done in class. Do **not** use the fact that $\mathbf{T}(t)\perp\mathbf{N}(t)$, and do **not** eliminate the parameter. Messy and/or undisciplined work may not be graded! (27 points)

Note: When differentiating, avoid using the Product Rule as an alternative to the Quotient Rule, unless you simplify your result to the most compact form!

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6) Assume that **r** is a position VVF of t in 3-space that is twice differentiable everywhere (i.e., second derivatives exist for all real t). Write a curvature formula we discussed for $\kappa(t)$ that involves a cross product. (4 points)

7) A helical curve C is determined by $\mathbf{r}(t) = \langle 2\cos t,\ 2\sin t,\ 3t \rangle$. The curvature at every point on the curve is given by a constant, κ. Find κ. Use your formula from Problem 6), and simplify your answer completely. Show all work! (16 points)

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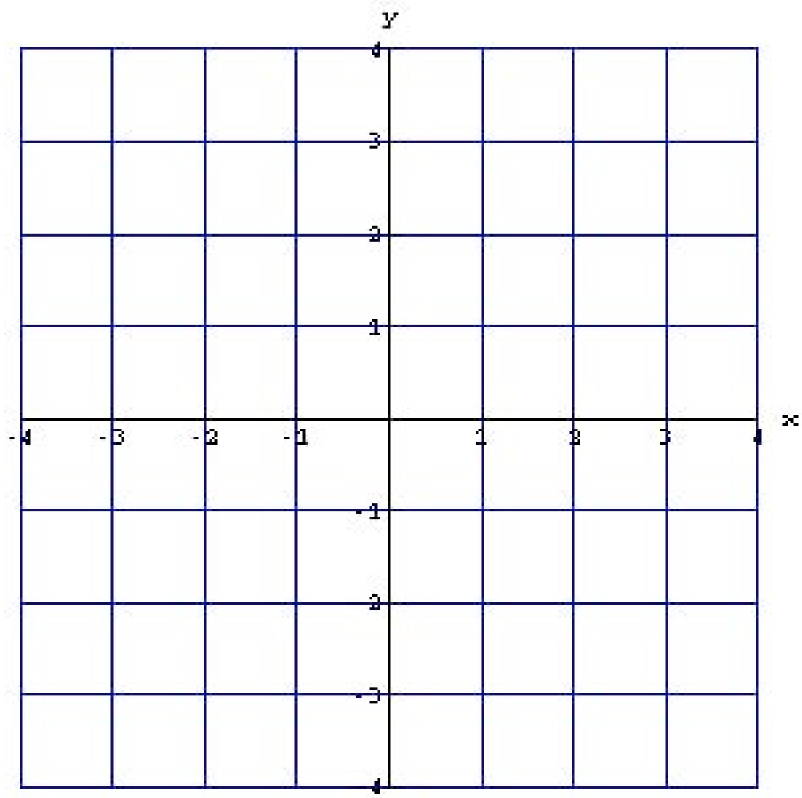
8) Sketch the level curves of $f(x,y) = (x-1)^2 + y^2$ for $k = 1, 4, 9$ on the grid below. Label the curves with their corresponding k-values. Be reasonably accurate. (8 points)



9) What is the graph of $z = (x-1)^2 + y^2$ in xyz-space? Problem 8) may help. Box in one: (3 points)

A Cone　　　　A Paraboloid　　　　A Sphere

10) Matching. (9 points total)

Fill in each blank with the best choice in the list below to indicate the level surface of f for the given value of k.

A. A Sphere or Ellipsoid
B. A Hyperboloid of One Sheet
C. A Hyperboloid of Two Sheets
D. A Cone
E. A Circular or Elliptic Paraboloid
F. A Hyperbolic Paraboloid
G. A Right Circular or Elliptic Cylinder
H. A Plane
I. A Line (a "degenerate" surface)
J. A Point (a "degenerate" surface)
K. NONE (no surface)

a) The level surface of $f(x,y,z)=2x-4y+5z,\ k=10$ is _____.

b) The level surface of $f(x,y,z)=x^2+y^2-z^2,\ k=4$ is _____.

c) The level surface of $f(x,y,z)=x^2+y^2-z^2,\ k=-4$ is _____.

11) Show that $\lim_{(x,y)\to(0,0)} \frac{2x^3 + y^3}{5x^3 - 2y^3}$ does not exist. (10 points)

12) Use polar coordinates to find $\lim_{(x,y)\to(0,0)} \frac{\sin\left(x^2 + y^2\right)}{x^2 + y^2}$. (5 points)