Math 252 Name: ________________________

QUIZ 2 (CHAPTER 15, 16.1, 16.2)

MATH 252 – FALL 2008 – KUNIYUKI
SCORED OUT OF 125 POINTS ⇒ MULTIPLIED BY 0.84 ⇒ 105% POSSIBLE

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

Clearly mark vectors, as we have done in class. I will use boldface, but you don't! When describing vectors or vector-valued functions, you may use either $\langle \; \rangle$ or "$\mathbf{i} - \mathbf{j} - \mathbf{k}$" notation. Assume we are in our usual 2- and 3-dimensional Cartesian coordinate systems. Give exact answers, unless otherwise specified.

USE THE BACK OF THIS TEST IF YOU NEED MORE SPACE!!

1) Find the length of the curve parameterized by:

$$x = t\sin t + \cos t, \quad y = t\cos t - \sin t, \quad z = \frac{3}{2}t^2, \quad 0 \le t \le 4.$$

Leave your answer as a simplified exact answer; do not approximate it using a calculator. Distance is measured in meters. Show all work! (20 points)

<u>YOU MAY CONTINUE ON THE BACK OF THIS SHEET.</u>

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2) Find parametric equations for the tangent line to C at the point $(25, 0, 478)$, where C is parameterized by: $x = t^3 - 2, \quad y = \sin(\pi t), \quad z = 2t^5 - t^2 + 1$.
(12 points)

3) A curve C in 3-space is smoothly parameterized by the position vector-valued function (VVF) rule $\mathbf{r}(t)$. The position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ are orthogonal for all real t. Simplify $D_t\left[\mathbf{r}(t) \bullet \mathbf{r}(t)\right]$ for all real t. Use a differentiation rule discussed in class. (5 points)

4) The acceleration of a moving particle is given by $\mathbf{a}(t)=(3\sin t)\mathbf{i}+(5\cos t)\mathbf{j}$. Find the position vector-valued function (VVF rule) $\mathbf{r}(t)$ if $\mathbf{r}(0)=2\mathbf{i}-6\mathbf{j}$ and $\mathbf{v}(0)=\mathbf{i}+3\mathbf{j}$. (15 points)

5) The curve C is determined by $\mathbf{r}(t) = \left\langle \frac{t^3}{3}, \frac{t^2}{2} \right\rangle$, where $t > 0$. Show all work, simplify radicals, and simplify completely, as we have done in class. Do **not** eliminate the parameter. Messy and/or undisciplined work may not be graded!

Note: When differentiating, avoid using the Product Rule as an alternative to the Quotient Rule, unless you simplify your result to the most compact form! (18 points total)

a) Find the unit tangent VVF (rule) $\mathbf{T}(t)$ for C.

b) Find the VVF (rule) $\mathbf{T}'(t)$ for C.

6) Assume that **r** is a position VVF of t in 3-space that is twice differentiable everywhere (i.e., second derivatives exist for all real t). Write a curvature formula we discussed for $\kappa(t)$ that involves a cross product. (4 points)

7) A twisted cubic curve C is determined by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$, where $t > 0$. (19 points total)

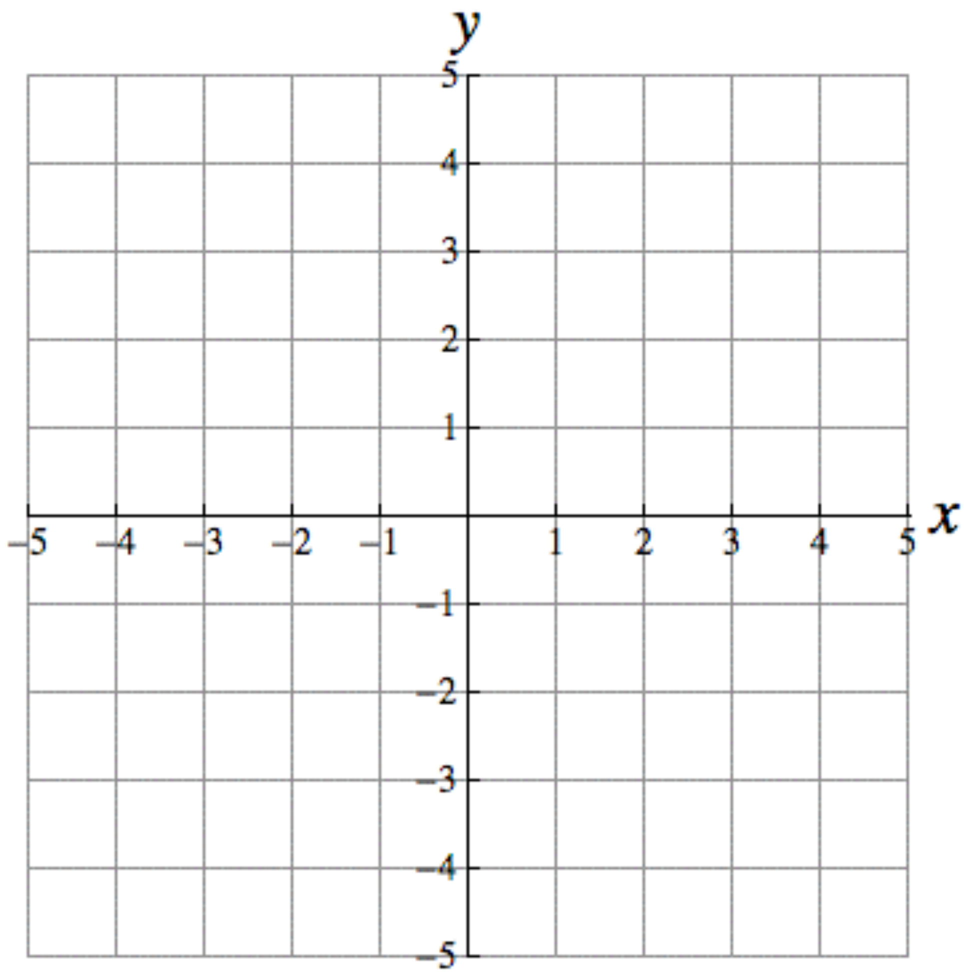
   a) Find a general curvature formula, $\kappa(t)$, for every point on C. Use your formula from Problem 6), and simplify your answer completely. Show all work! (15 points)

<u>YOU MAY CONTINUE ON THE BACK OF THIS SHEET. DO PART b).</u>

a) cont.)

b) Use your formula in part a) to find the curvature of the twisted cubic curve C at the point $(2, 4, 8)$. Approximate your answer to four decimal places. (4 points)

8) Sketch the level curves of $f(x, y) = y - x^2$ for $k = -3, 0, 3$ on the grid below. Label the curves with their corresponding *k*-values. Be reasonably accurate. (8 points)



9) Matching. (9 points total)

Fill in each blank with the best choice (A-K) in the list below to indicate the level surface of f for the given value of k.

A. A Sphere or Ellipsoid
B. A Hyperboloid of One Sheet
C. A Hyperboloid of Two Sheets
D. A Cone
E. A Circular or Elliptic Paraboloid
F. A Hyperbolic Paraboloid
G. A Right Circular or Elliptic Cylinder
H. A Plane
I. A Line (a “degenerate” surface)
J. A Point (a “degenerate” surface)
K. NONE (no surface)

a) The level surface of $f(x,y,z) = x^2 + z^2$, $k = 7$ is _____.

b) The level surface of $f(x,y,z) = 3x + 4y - 5z$, $k = 0$ is _____.

c) The level surface of $f(x,y,z) = x^2 + 4y^2 + 9z^2$, $k = 25$ is _____.

10) Show that $\lim_{(x,y)\to(0,0)} \frac{3x^2 - 4y^2}{2x^2 + 3y^2}$ does not exist. (8 points)

11) Use polar coordinates to find $\lim_{(x,y)\to(0,0)} \frac{x^3 y^3}{x^2 + y^2}$. (7 points)