Math 252 Name: ________________________

QUIZ 3 (SECTIONS 16.3-16.9)

MATH 252 – FALL 2007 – KUNIYUKI
SCORED OUT OF 125 POINTS ⇒ MULTIPLIED BY 0.84 ⇒ 105% POSSIBLE

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

USE THE LAST PAGE IF YOU NEED MORE SPACE!!

1) Let $f(x,y,z)=\sqrt{3x^2y+z^3}$. Find $f_x(x,y,z)$. (4 points)

2) Let $f(r,s)=\cos(rs)$. Find $f_r(r,s)$ and use that to find $f_{rs}(r,s)$. (6 points)

3) Assume that f is a function of x and y. Write the limit definition of $f_y(x,y)$ using the notation from class. (4 points)

4) Let $f(x,y)=3xy^2-4y^3+5$. Use differentials to linearly approximate the change in f if (x,y) changes from $(4,-2)$ to $(3.98,-1.96)$. (12 points)

5) Let f, f_1, f_2 and f_3 be differentiable functions such that $w = f(x, y, z)$, $x = f_1(u, v)$, $y = f_2(u, v)$, and $z = f_3(u, v)$. Use the Chain Rule to write an expression for $\frac{\partial w}{\partial v}$. (5 points)

6) Find $\frac{\partial z}{\partial x}$ if $z = f(x, y)$ is a differentiable function described implicitly by the equation $e^{xyz} = xz^4$. Use the Calculus III formula given in class. Simplify. (9 points)

7) The temperature at any point (x, y) in the *xy*-plane is given by $f(x, y) = 2xy + x^2$ in degrees Fahrenheit. Assume x and y are measured in meters. Give units in your answers. (23 points total)

a) Find the **maximum** rate of change of temperature at the point $(3, 4)$. Approximate your final answer to three significant digits.

7 continued)

Reminder: The temperature at any point (x, y) in the *xy*-plane is given by $f(x, y) = 2xy + x^2$ in degrees Fahrenheit. Assume x and y are measured in meters. Give units in your answers.

b) Find the rate of change of temperature at $(3, 4)$ in the direction of $\mathbf{i} - 3\mathbf{j}$. Approximate your final answer to three significant digits.

7 continued)

c) Find a non-**0** direction vector **v** such that the rate of change of temperature at $(3, 4)$ in the direction of **v** is 0 [units].

8) Find an equation for the tangent plane to the graph of the equation $5x^2 - 4y^2 + z^2 = 45$ at the point $P(-3, 2, 4)$. (10 points)

9) Find all critical points of $f(x,y) = 2x^3 + 2y^3 + 3y^2 - 54x - 12y + 1$, and classify each one as a local maximum, a local minimum, or a saddle point. Show all work, as we have done in class. You do not have to find the corresponding function values. (27 points)

YOU MAY CONTINUE ON THE BACK OF THIS SHEET.

(blank page)

10) Find the absolute maximum and absolute minimum of $f(x, y, z) = 3x - 2y + 5z$ subject to the constraint $9x^2 + y^2 + \frac{15}{2}z^2 = 1$ using the method of Lagrange multipliers. Your answers will be ordered triples in the domain of f; label which one corresponds to the absolute maximum and which one corresponds to the absolute minimum. Give exact, simplified answers, and rationalize denominators. (25 points)