Math 252 Name: ________________________

QUIZ 3 (SECTIONS 16.3-16.9)

MATH 252 – FALL 2008 – KUNIYUKI

SCORED OUT OF 125 POINTS ⇒ MULTIPLIED BY 0.84 ⇒ 105% POSSIBLE

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

USE THE BLANK SHEET AT THE END IF YOU NEED MORE SPACE!!

1) Let $f(r,t)=t^2\sin\left(\frac{r}{t}\right)$. Find $f_r(r,t)$. (5 points)

2) Let $f(x, y, z) = e^{xyz}$. Find $f_y(x, y, z)$ and use that to find $f_{yz}(x, y, z)$. (8 points)

3) Assume that f is a function of s and t. Write the limit definition of $f_s(s, t)$ using the notation from class. (4 points)

4) Find $\dfrac{\partial z}{\partial y}$ if $z = f(x,y)$ is a differentiable function described implicitly by the equation $\tan\left(y^3 z\right) = x^2 - yz$. Use the Calculus III formula given in class. Simplify. (12 points)

5) Let f, g, and h be differentiable functions such that $z = f(u, v)$, $u = g(r, s, t)$, and $v = h(r, s, t)$. Use the Chain Rule to write an expression for $\frac{\partial z}{\partial t}$. (5 points)

6) The temperature at any point (x, y) in the *xy*-plane is given by $f(x, y) = 5x^2 y + y^3$ in degrees Fahrenheit. Assume *x* and *y* are measured in meters. Give appropriate units in your answers. (30 points total)

   a) Find the **maximum** rate of change of temperature at the point $(2, -3)$. Approximate your final answer to four significant digits.

6 continued)

Reminder: The temperature at any point (x, y) in the *xy*-plane is given by $f(x, y) = 5x^2 y + y^3$ in degrees Fahrenheit. Assume x and y are measured in meters. Give appropriate units in your answers.

b) Find the rate of change of temperature at $(2, -3)$ in the direction of $4\mathbf{i} + \mathbf{j}$. Approximate your final answer to four significant digits.

Reminder: The temperature at any point (x, y) in the *xy*-plane is given by $f(x, y) = 5x^2 y + y^3$ in degrees Fahrenheit. Assume *x* and *y* are measured in meters. Give appropriate units in your answers.

c) Use differentials to linearly approximate the change in temperature if (x, y) changes from $(2, -3)$ to $(2.03, -3.01)$.

d) In what direction does the temperature **decrease** most rapidly at $(2, -3)$? Give an appropriate non-**0** direction vector.

7) Find an equation for the tangent plane to the graph of the equation $z^3 = x - xy$ at the point $P(8, -26, 6)$. (10 points)

8) Find all critical points of $f(x, y) = 2x^4 + y^2 - 12xy$, and classify each one as a local maximum, a local minimum, or a saddle point. Show all work, as we have done in class. You do not have to find the corresponding function values.
(26 points)

YOU MAY USE THE BLANK SHEET AT THE END!

9) The function f, where $f(x,y,z)=(x-3)^2+(y-5)^2+(z-7)^2$, gives the squared distance of the point (x,y,z) from the point $(3,5,7)$ in *xyz*-space. Using the method of Lagrange multipliers, find the point on the unit sphere $x^2+y^2+z^2=1$ that is closest to the point $(3,5,7)$. The point will lie in Octant I; you may use this fact without proof. You do <u>not</u> have to prove that your candidate point corresponds to an absolute minimum of f under the constraint. Give an exact, simplified answer, and rationalize denominators. (25 points)