Math 252 Name: ________________________

# QUIZ 4 (CHAPTER 17)

**MATH 252 – FALL 2007 – KUNIYUKI**
**SCORED OUT OF 125 POINTS ⇒ MULTIPLIED BY 0.84 ⇒ 105% POSSIBLE**

**Show all work, simplify as appropriate, and use "good form and procedure" (as in class).**
**Box in your final answers!**
**No notes or books allowed. A scientific calculator is allowed.**

1) Reverse the order of integration, and **evaluate** the resulting double integral: $\int_0^{27} \int_{\sqrt[3]{x}}^{3} \cos\left(1+y^4\right) dy\, dx$. Give a simplified exact answer; do not approximate. Sketch the region of integration. (20 points)

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2) Let $R$ be the region in the $xy$-plane that is bounded by the rectangle with vertices $(1,3)$, $(7,3)$, $(7,5)$, and $(1,5)$. **Set up a double integral** for the surface area of the portion of the graph of $9x^2+4y^2+z^2=1000$ $(z>0)$ that lies over $R$. Make sure your double integral is as detailed as possible; do not leave in generic notation like $R$ or $f$. Also, do not leave $dA$ in your final answer; break it down into $d(\text{variable})\,d(\text{variable})$. Do **not** evaluate. (14 points)

3) Find the **center of mass** of the lamina that makes up the region in the *xy*-plane bounded by the *x*-axis and the graph of $y = 4 - x^2$. The area mass density is given by $\delta(x, y) = 3y$. Sketch the region. Give exact coordinates; do not approximate. Hint: You may use the following rule based on the Binomial Theorem: $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$. (32 points)

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4) The point $P$ has cylindrical coordinates $\left( r = 7,\ \theta = \dfrac{\pi}{3},\ z = 2 \right)$.

Find the Cartesian coordinates of point $P$. Fill in the blanks below.
Give exact answers; do not approximate. (9 points)

$x =$ _______

$y =$ _______

$z =$ _______

5) The point $Q$ has spherical coordinates $\left( \rho = 3,\ \phi = \dfrac{\pi}{3},\ \theta = \dfrac{\pi}{6} \right)$.

Find the Cartesian coordinates of point $Q$. Fill in the blanks below.
Give exact answers; do not approximate. (12 points)

$x =$ _______

$y =$ _______

$z =$ _______

6) Find the **mass** of a solid hemisphere of radius *a* lying on its base in terms of the constant of proportionality, *k*. Assume that the mass density at any point in the solid is directly proportional to the distance from the center of the base to the point. Use spherical coordinates to complete the problem. (18 points)

7) Use the Jacobian-based change of variables method to evaluate $\iint_R \left(9x^2+16y^2\right)dA$, where $R$ is the region in the $xy$-plane bounded by the graph of $9x^2+16y^2=36$. Use the change of variables: $x=\frac{u}{3}$ and $y=\frac{v}{4}$.

Hint: You will eventually want to go to polar coordinates.

Give a simplified exact answer; you do not have to approximate it. (20 points)

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