Math 252 Name: ________________________

QUIZ 4 (CHAPTER 17)

MATH 252 – FALL 2008 – KUNIYUKI

SCORED OUT OF 125 POINTS ⇒ MULTIPLIED BY 0.84 ⇒ 105% POSSIBLE

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

1) Reverse the order of integration, and **evaluate** the resulting double integral:
$\int_0^{16}\int_{\sqrt[4]{y}}^{2} \frac{y}{\sqrt{113+x^9}}\,dx\,dy$. Give a simplified exact answer; do not approximate.
Sketch the region of integration; you may use different scales for the x- and y-axes. (20 points). **YOU MAY CONTINUE ON THE BACK.**

2) Evaluate $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} e^{3x^2+3y^2}\, dy\, dx$. Your answer must be exact and simplified. You do not have to approximate it. (16 points).

<u>YOU MAY CONTINUE ON THE BACK.</u>

3) Find the surface area of the portion of the graph of $z = 7x + \frac{1}{2}y^2$ that lies over R, where R is the region in the xy-plane bounded by the rectangle with vertices $(0, 0)$, $(4, 0)$, $(0, 2)$, and $(4, 2)$. Give an exact answer; you do not have to approximate it. Distance is measured in meters.

Major Hint: Use the following Table of Integrals formula:

$$\int \sqrt{a^2 + u^2}\, du = \frac{u}{2}\sqrt{a^2 + u^2} + \frac{a^2}{2}\ln\left| u + \sqrt{a^2 + u^2} \right| + C$$

(20 points). **YOU MAY CONTINUE ON THE BACK.**

4) **Set up a triple integral** for the volume of the solid bounded by the graphs of $y = x^2 - 3x$, $2x - y = 0$, $z = 5$, and $z - e^x - e^y = 5$. Make sure your triple integral is as detailed as possible; do not leave in generic notation like R or f. Also, do not leave dV in your final answer; break it down into $d(\text{variable})\,d(\text{variable})\,d(\text{variable})$. Do **not** evaluate. (15 points)

<u>YOU MAY CONTINUE ON THE BACK.</u>

5) A solid is bounded by the graph of $z = x^2 + y^2$, the graph of $x^2 + y^2 = 16$, and the *xy*-plane. The density at any point in the solid is directly proportional to the square of the distance from the point to the *xy*-plane. Find the center of mass of the solid; it turns out to be a point outside of the actual solid. Use cylindrical coordinates to complete the problem. If you use any short cuts, explain them precisely! Warning: You may run into some very big numbers along the way! (29 points). **YOU MAY CONTINUE ON THE BACK AND ON THE NEXT SHEET.**

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6) Consider the Cartesian (or rectangular) coordinates *x*, *y*, and *z*, and the spherical coordinates ρ, ϕ, and θ. Verify that, for the spherical coordinate transformation, the Jacobian $\dfrac{\partial(x, y, z)}{\partial(\rho, \phi, \theta)} = \rho^2 \sin\phi$. Begin by expressing *x*, *y*, and *z* in terms of ρ, ϕ, and θ; you may do this from memory without showing work. Work out an appropriate determinant, and clearly show each step; do not simply use a geometric argument. (25 points).

<u>YOU MAY CONTINUE ON THE BACK.</u>