Math 252 Name: ________________________

QUIZ 5 (CHAPTER 18)

MATH 252 – FALL 2006 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

USE THE BACK OF THIS TEST IF YOU NEED MORE SPACE!!

1) C consists of the curves C_1 and C_2 in the *xy*-plane. That is, $C = C_1 \cup C_2$. The curve C_1 is the directed line segment from $(0, 0)$ to $(4, 2)$, and the curve C_2 is the portion of the parabola $x = y^2$ directed from $(4, 2)$ to $(9, 3)$. If the force at (x, y) is $\mathbf{F}(x, y) = \langle 4y^3,\ 3x \rangle$, find the work done by $\mathbf{F}$ along C. It is recommended that you write your final answer as a decimal. (25 points)

YOU MAY CONTINUE ON THE BACK.

2) Use the idea of potential functions and the Fundamental Theorem for Line Integrals to show that the following line integral is independent of path in Octant I of *xyz*-space and to evaluate the integral. Show all work, as we have done in class. Use good form. In particular, indicate independent variables for functions; for example, write $f(x, y, z)$ instead of simply f. Give an exact, simplified answer; do **not** approximate.

$$\int_{(2,1,1)}^{(1,2,3)} (6x-1)\,dx + \left(4e^{2z}\right) dy + \left(8ye^{2z} + \frac{1}{z}\right) dz$$

(27 points)

YOU MAY CONTINUE ON THE BACK.

3) Let $\mathbf{F}(x, y, z) = 3\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$. Let S be the portion of the “half” cone $z = \sqrt{x^2 + y^2}$ that is inside the cylinder $x^2 + y^2 = 4$ but outside the cylinder $x^2 + y^2 = 1$. Find the flux of **F** across S, given by $\iint_S \mathbf{F} \bullet \mathbf{n}\, dS$, where **n** is always taken to be the unit upper normal to S. (25 points)

YOU MAY CONTINUE ON THE BACK.

4) Use the Divergence Theorem to find the flux of $\mathbf{F}(x, y, z) = \langle \ln(yz), 7y, z \rangle$ through any sphere S of radius 3. (10 points)

5) Assume that the hypotheses of Stokes's Theorem (as stated in my 18.7 Notes) are satisfied. In particular, S has equation $z = f(x, y)$ and is a "capping surface" for a piecewise smooth simple closed curve C. Fill in the blank: (3 points)

According to Stokes's Theorem, $\oint_C \mathbf{F} \bullet \mathbf{T}\, ds =$ ______________________________ .

6) Let $\mathbf{F}(x, y, z) = \langle y^2 \sin x,\ 4e^y + y,\ x^3 yz^3 \rangle$. (15 points total)

a) Find div **F**.

b) Find **curl F**.