Math 252 Name: ________________________

QUIZ 5 (CHAPTER 18)

MATH 252 – FALL 2008 – KUNIYUKI
105 POINTS TOTAL, BUT 100 POINTS = 100%

Show all work, simplify as appropriate, and use "good form and procedure" (as in class).
Box in your final answers!
No notes or books allowed. A scientific calculator is allowed.

USE THE BACK OF THIS TEST IF YOU NEED MORE SPACE!!

1) Matching. (9 points total)

Fill in each blank below with a true property describing the vector field **F**. (Assume that we are only evaluating **F** on its domain.)

A. The vectors in the field all have the same direction.
B. The non-**0** vectors in the field all point away from the origin.
C. The vectors in the field are all unit vectors.

I. $\mathbf{F}(x, y) = x\mathbf{i} + y\mathbf{j}$. It is true that _____.

II. $\mathbf{F}(x, y) = 2\mathbf{i} + 3\mathbf{j}$. It is true that _____.

III. $\mathbf{F}(x, y) = \dfrac{1}{\sqrt{x^2 + y^2}}(-x\mathbf{i} - y\mathbf{j})$. It is true that _____.

2) Let $\mathbf{F}(x, y, z) = \left\langle x^2 e^{2z},\ \cos(3y),\ xy^2 z^3 - x \right\rangle$. (20 points total)

a) Find div $\mathbf{F}$.

b) Find **curl F**.

3) C consists of the curves C_1 and C_2 in xyz-space. That is, $C = C_1 \cup C_2$.
The curve C_1 is the directed line segment from $(0, 2, 3)$ to $(2, 8, 4)$, and the curve C_2 is the portion of the graph of $y = x^3$ in the plane $z = 4$ directed from $(2, 8, 4)$ to $(3, 27, 4)$. If the force at (x, y, z) is $\mathbf{F}(x, y, z) = \langle xy,\ z + 4,\ 3 \rangle$, find the work done by **F** along C. It is recommended that you write your final answer as a decimal. Hint: If you want, you can analyze C_2 first. (32 points)

YOU MAY CONTINUE ON THE BACK.

4) Find the exact mass of a thin wire C in xyz-space if the linear mass density at any point (x, y, z) where $x \geq 0$ is given by $\delta(x, y, z) = 5x$ (i.e., five times the point's distance from the yz-plane), and if C is parameterized by $x = 3\cos t$, $y = 3\sin t$, and $z = 7t$, where $0 \leq t \leq \dfrac{\pi}{4}$. (17 points)

5) Use the idea of potential functions and the Fundamental Theorem for Line Integrals to show that the following line integral is independent of path in *xyz*-space and to evaluate the integral. Show all work, as we have done in class. Use good form. In particular, indicate independent variables for functions; for example, write $f(x,y,z)$ instead of simply f. Give an exact, simplified answer; do **not** approximate.

$$\int_{(-1,0,2)}^{(3,4,1)} \left(4y^2 + z\right) dx + \left(8xy - 3ze^{3y}\right) dy + \left(x - e^{3y} + 3z^2\right) dz$$

(27 points)

YOU MAY CONTINUE ON THE NEXT SHEET.