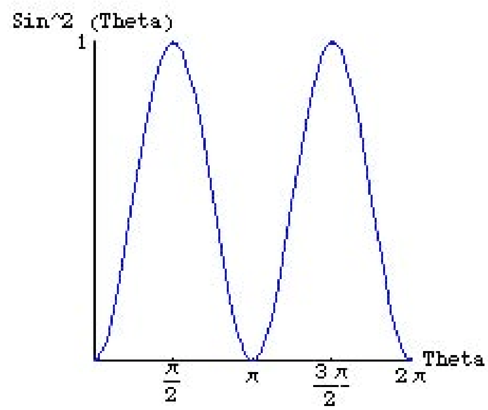
How do you graph the surface $z = \sin^2 \theta$ over the annulus R from our example in the 17.3 notes? How would you graph the corresponding solid whose volume we were finding?

First off, let's consider the graph of $\sin^2 \theta$ vs. θ in Cartesian coordinates:

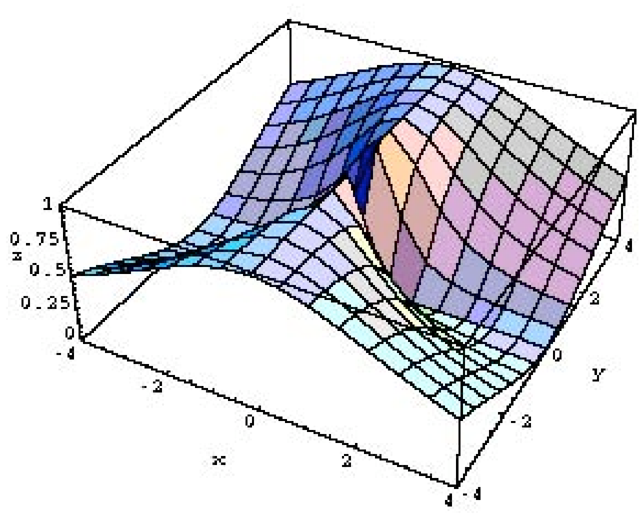


The square of a real number in [0,1] will also be in [0,1].
Because the range of $\sin\theta$ is [0,1], the range of $\sin^2 \theta$ is also [0,1].

Unlike the graph for $\left|\sin\theta\right|$, there are no corners at θ-values of 0, π, 2π, etc. The $\sin^2 \theta$ function is everywhere differentiable! Its derivative is given by $2\sin\theta\cos\theta$, or $\sin(2\theta)$, which is 0 at $\theta = \pi n$ $(n \text{ integer})$.

How do you graph $z = \sin^2 \theta$ in 3-space?

Here's what *Mathematica* gives; the coordinate axes are rotated a bit differently from what we're used to.

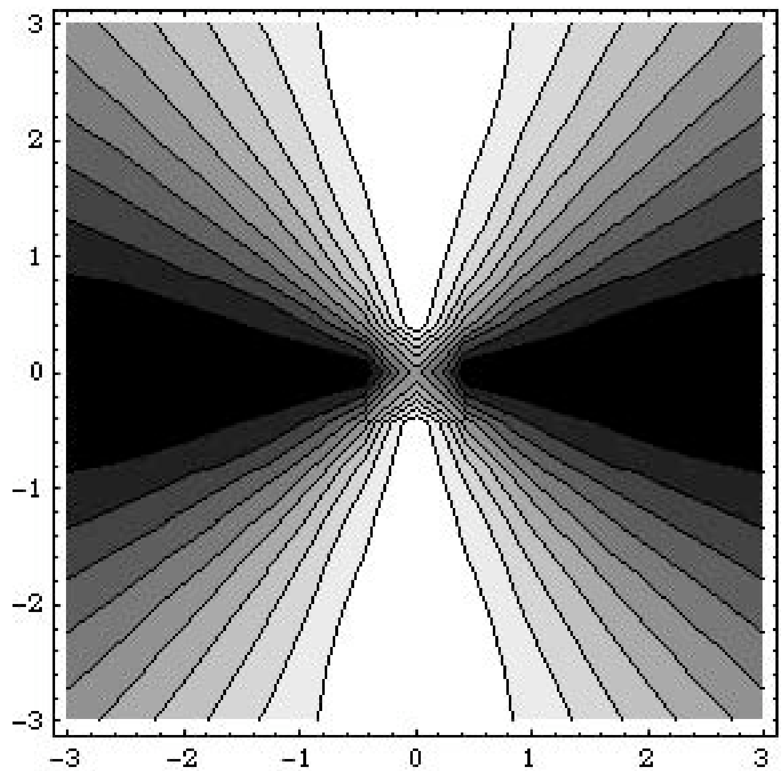


How do we get the piece over R?

You could take scissors to the previous graph, and rotate the scissors as you cut.

It's basically like a twisty slide, or a piece of a roller coaster. Imagine a staircase in a mansion that is curving upward, except that we smooth out the steps. Notice that the function values are constant along the straight lines in the xy-plane through the origin: when θ is fixed, it doesn't matter what r is. The steps are flat, because our function is independent of r and therefore doesn't care about r; it's like sweeping through r-values. The level curves are line segments pointing away from the origin, and they vary from an f or z value of 0 to a value of 1.

Mathematica gives the following Contour Plot; ignore the curviness of some of the lines – these are distortions.



When $\theta = 0$, for example, $\sin^2\theta$ = 0, all the way from $r = 2$ to $r = 3$. That means that the line segment from (2,0,0) to (3,0,0) in Cartesian coordinates is going to be on our slide / staircase. In fact, it will be the bottom edge of our staircase.

When $\theta = \frac{\pi}{2}$, $\sin^2\theta$ = 1, all the way from $r = 2$ to $r = 3$. That means that the line segment from (0,2,1) to (0,3,1) in Cartesian coordinates is going to be on our slide. In fact, it will be the top edge of our staircase.

As θ increases from 0 to $\frac{\pi}{2}$, $\sin^2\theta$ increases from 0 to 1 in a curvy way, like the way we discussed in class. $\sin^2\theta$ gives us the *z*-coordinate of our step on the staircase. We don't get any hills or valleys along the staircase, though, because $\sin^2\theta$ is always increasing between $\theta = 0$ and $\theta = \frac{\pi}{2}$. If we go beyond $\frac{\pi}{2}$, though, then the staircase begins to go down.

Look at the first *Mathematica* graph. The top edge of the staircase lies on that top "crease", though it's not really a sharp crease (no corner in our first graph!). The solid whose volume we're finding is basically the wall beneath the staircase.