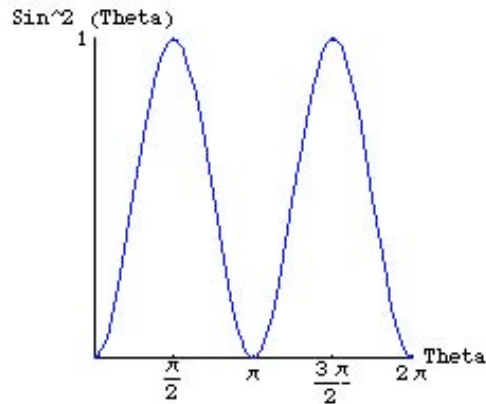


How do you graph the surface $z = \sin^2 \theta$ over the annulus R from our example in the 17.3 notes? How would you graph the corresponding solid whose volume we were finding?

First off, let's consider the graph of $\sin^2 \theta$ vs. θ in Cartesian coordinates:

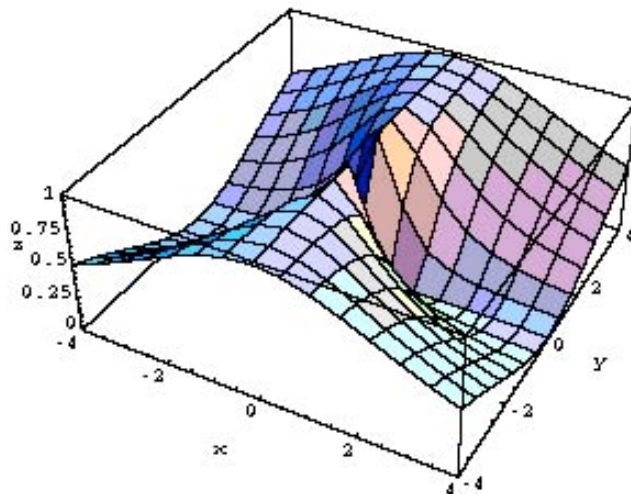


The square of a real number in $[0,1]$ will also be in $[0,1]$.
 Because the range of $\sin \theta$ is $[0,1]$, the range of $\sin^2 \theta$ is also $[0,1]$.

Unlike the graph for $|\sin \theta|$, there are no corners at θ -values of $0, \pi, 2\pi$, etc. The $\sin^2 \theta$ function is everywhere differentiable! Its derivative is given by $2 \sin \theta \cos \theta$, or $\sin(2\theta)$, which is 0 at $\theta = \pi n$ (n integer).

How do you graph $z = \sin^2 \theta$ in 3-space?

Here's what *Mathematica* gives; the coordinate axes are rotated a bit differently from what we're used to.

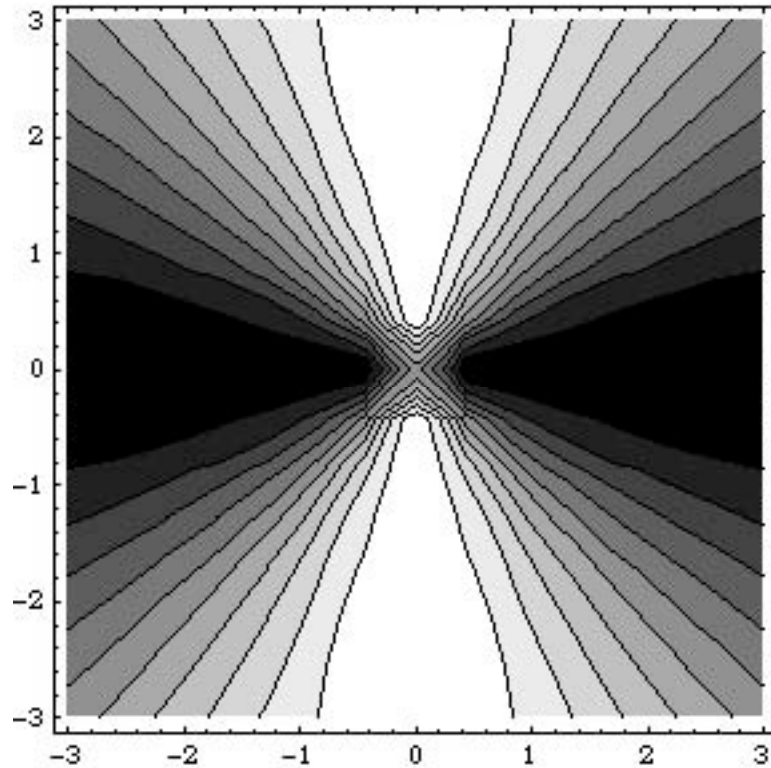


How do we get the piece over R ?

You could take scissors to the previous graph, and rotate the scissors as you cut.

It's basically like a twisty slide, or a piece of a roller coaster. Imagine a staircase in a mansion that is curving upward, except that we smooth out the steps. Notice that the function values are constant along the straight lines in the xy -plane through the origin: when θ is fixed, it doesn't matter what r is. The steps are flat, because our function is independent of r and therefore doesn't care about r ; it's like sweeping through r -values. The level curves are line segments pointing away from the origin, and they vary from an f or z value of 0 to a value of 1.

Mathematica gives the following Contour Plot; ignore the curviness of some of the lines – these are distortions.



When $\theta = 0$, for example, $\sin^2 \theta = 0$, all the way from $r = 2$ to $r = 3$. That means that the line segment from $(2,0,0)$ to $(3,0,0)$ in Cartesian coordinates is going to be on our slide / staircase. In fact, it will be the bottom edge of our staircase.

When $\theta = \frac{\pi}{2}$, $\sin^2 \theta = 1$, all the way from $r = 2$ to $r = 3$. That means that the line segment from $(0,2,1)$ to $(0,3,1)$ in Cartesian coordinates is going to be on our slide. In fact, it will be the top edge of our staircase.

As θ increases from 0 to $\frac{\pi}{2}$, $\sin^2 \theta$ increases from 0 to 1 in a curvy way, like the way we discussed in class. $\sin^2 \theta$ gives us the z -coordinate of our step on the staircase. We don't get any hills or valleys along the staircase, though, because $\sin^2 \theta$ is always increasing between $\theta = 0$ and $\theta = \frac{\pi}{2}$. If we go beyond $\frac{\pi}{2}$, though, then the staircase begins to go down.

Look at the first *Mathematica* graph. The top edge of the staircase lies on that top "crease", though it's not really a sharp crease (no corner in our first graph!). The solid whose volume we're finding is basically the wall beneath the staircase.