We need a point on the desired plane. Three immediate choices are:

$(3, 1, -5)$, which is on line l_1,
$(-2, 3, 8)$, which is on line l_2, and
$(10, -13, 16)$, which is the intersection point we found in part a).

Let's use $\mathbf{n} = \langle 8, 7, 2 \rangle$ as our normal and $(3, 1, -5)$, say, as our point.

Standard form for an equation of the plane:

$$\boxed{8(x-3)+7(y-1)+2(z-(-5))=0}$$

General form for an equation of the plane:

$$\boxed{8x+7y+2z-21=0}$$

9) Consider the point $P(7, 2, -1)$ and the plane $4x-3y+2z+60=0$. Distance is measured in meters. (16 points total)

a) At what point does the given plane intersect the x-axis? (We will call this point Q.)

Along the x-axis, $y=0$ and $z=0$, so we substitute $y=0$ and $z=0$ in the given equation and solve for x:

$$\begin{aligned} 4x-3y+2z+60=0, \quad y=0, \quad z=0 \quad &\Rightarrow \\ 4x-3(0)+2(0)+60&=0 \\ 4x+60&=0 \\ x&=-15 \end{aligned}$$

The desired point is: $\boxed{Q(-15, 0, 0).}$

b) Find a normal vector for the given plane. (We will call this vector $\mathbf{n}$.)

$$\boxed{\mathbf{n} = \langle 4, -3, 2 \rangle.}$$

c) If we let the vector $\mathbf{p} = \overrightarrow{QP}$, then the distance between the given point P and the given plane equals: $\left|\text{comp}_{\mathbf{n}}\mathbf{p}\right|$. Use the component formula to find $\left|\text{comp}_{\mathbf{n}}\mathbf{p}\right|$. Round it off to the nearest tenth of a meter.

Find the **p** vector:

$$\begin{aligned}\mathbf{p} &= \overrightarrow{QP} \\ &= \langle 7-(-15), 2-0, -1-0\rangle \\ &= \langle 22, 2, -1\rangle\end{aligned}$$

The desired distance is:

$$\begin{aligned}\left|\text{comp}_{\mathbf{n}}\mathbf{p}\right| &= \frac{|\mathbf{p}\bullet\mathbf{n}|}{\|\mathbf{n}\|} \\ &= \frac{\left|\langle 22, 2, -1\rangle\bullet\langle 4, -3, 2\rangle\right|}{\|\langle 4, -3, 2\rangle\|} \\ &= \frac{|88-6-2|}{\sqrt{(4)^2+(-3)^2+(2)^2}} \\ &= \frac{80}{\sqrt{29}} \\ &\approx \boxed{14.9 \text{ meters}}\end{aligned}$$

Observe that we would have gotten the same answer from the shortcut formula given in Section 14.5 for a point (x_0, y_0, z_0) and a plane $ax+by+cz+d=0$:

$$\begin{aligned}h &= \frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}} \\ &= \frac{|4(7)-3(2)+2(-1)+60|}{\sqrt{(4)^2+(-3)^2+(2)^2}} \\ &= \frac{80}{\sqrt{29}} \\ &\approx 14.9 \text{ meters}\end{aligned}$$

10) Matching. (12 points total)

Fill in each blank below with one of the following:

A. An Ellipsoid
B. A Hyperboloid of One Sheet
C. A Hyperboloid of Two Sheets
D. A Cone
E. An Elliptic Paraboloid
F. A Hyperbolic Paraboloid

I. The graph of $\frac{1}{2}x^2 - 3y^2 - z^2 = 5$ is __**C**__.

(Think: $x^2 - y^2 - z^2 = 1$.)

II. The graph of $x^2 + 7y^2 - z = 0$ is __**E**__.

(Think: $x^2 + y^2 - z = 0$, or $z = x^2 + y^2$.)

III. The graph of $4x^2 - 9y^2 + z^2 = 0$ is __**D**__.

(Think: $x^2 - y^2 + z^2 = 0$, or $y^2 = x^2 + z^2$.)

IV. The graph of $4x^2 - y^2 + 11z^2 = 7$ is __**B**__.

(Think: $x^2 - y^2 + z^2 = 1$.)

11) Consider the graph of $4x^2 - y^2 + 11z^2 = 7$. This was in Problem 10, part IV. Assume that k takes the place of real constants. (12 points total)

The graph is a hyperboloid of one sheet. For simplicity, consider: $x^2 - y^2 + z^2 = 1$

a) The axis of the graph is the … (Box in one:)

x-axis $\boxed{y\text{-axis}}$ z-axis

Observe that y is the "odd man out" in $x^2 - y^2 + z^2 = 1$.

b) The conic traces of the graph in the planes $x = k$ are … (Box in one:)

Ellipses $\boxed{\text{Hyperbolas}}$ Parabolas

Let k be a real number such that $|k| > 1$. The trace of the graph of $x^2 - y^2 + z^2 = 1$ in the plane $x = k$ is given by:

$$\begin{aligned} k^2 - y^2 + z^2 &= 1, & x &= k \\ k^2 - 1 &= y^2 - z^2, & x &= k \\ y^2 - z^2 &= \underbrace{k^2 - 1}_{>0}, & x &= k \end{aligned}$$

We obtain hyperbolas.

c) The conic traces of the graph in the planes $y = k$ are … (Box in one:)

$\boxed{\text{Ellipses}}$ Hyperbolas Parabolas

Let k be a real number. The trace of the graph of $x^2 - y^2 + z^2 = 1$ in the plane $y = k$ is given by:

$$\begin{aligned} x^2 - k^2 + z^2 &= 1, & y &= k \\ x^2 + z^2 &= \underbrace{1 + k^2}_{>0}, & y &= k \end{aligned}$$

This is a circle, but we have ellipses for the traces of the graph of the original equation because of the deformations produced by the coefficients.

The ellipse family of traces makes sense, since planes of the form $y = k$ are perpendicular to the axis of the graph of $x^2 - y^2 + z^2 = 1$, which is a hyperboloid of one sheet.

d) The conic traces of the graph in the planes $z = k$ are … (Box in one:)

Ellipses $\boxed{\text{Hyperbolas}}$ Parabolas

Let k be a real number such that $|k| > 1$. The trace of the graph of $x^2 - y^2 + z^2 = 1$ in the plane $z = k$ is given by:

$$\begin{aligned} x^2 - y^2 + k^2 &= 1, & z &= k \\ k^2 - 1 &= y^2 - x^2, & z &= k \\ y^2 - x^2 &= \underbrace{k^2 - 1}_{>0}, & z &= k \end{aligned}$$

We obtain hyperbolas.

12) Find an equation (in x, y, and z) of the surface obtained by revolving the graph of $4y^2 + 25z^2 = 1$ (in the yz-plane) about the z-axis. (3 points)

Since y is the "non-axis" variable in the equation above, and x is the "missing variable" …

We replace y^2 with $(x^2 + y^2)$. We don't "touch z."

$$4(x^2 + y^2) + 25z^2 = 1$$

$$\boxed{4x^2 + 4y^2 + 25z^2 = 1}$$

We are taking an ellipse in the yz-plane, and we are using it to generate an ellipsoid.