Math 254 Name: ________________________

FINAL

MATH 254 - SUMMER 2002 - KUNIYUKI

GRADED OUT OF 100 POINTS × 2 = 200 POINTS TOTAL

Circle your final answers!

Show all work and simplify wherever appropriate, as we have done in class!

A scientific calculator and an appropriate sheet of notes are allowed on this exam.

Assume that n represents a positive integer.

1) Find A^{10} if $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$. (3 points)

2) Find the determinant below. (10 points)

$$\begin{vmatrix} 2 & 3 & 3 & 0 & 1 \\ 1 & -1 & -2 & 0 & 4 \\ 0 & 5 & 0 & 0 & 0 \\ 3 & 4 & 1 & 3 & -2 \\ 1 & 0 & 2 & 0 & 1 \end{vmatrix}$$

3) If A is a 4×4 matrix, and $|A| = 10$, then find $|3A|$. (3 points)

4) Let $\mathbf{v}_1 = (0,3,6,0)$, $\mathbf{v}_2 = (0,2,4,6)$, and $\mathbf{v}_3 = (1,-1,-2,1)$.
Express $(4,-1,-2,-11)$ as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.
The weights in this linear combination must be given as specific real numbers; don't just leave them as "c_i"s. (12 points)

5) Find the angle between the vectors $(1,3,3)$ and $(2,1,-4)$. Round off your answer to the nearest tenth of a degree. (For maximum accuracy, don't round off until the end.) (8 points)

6) $T: R^4 \to R^3$ is a linear transformation such that $T(\mathbf{v}) = A\mathbf{v}$, where A is the matrix below. (6 points total; 2 points each)

$$A = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 0 & -5 \end{bmatrix}$$

a) What is the dimension of the kernel of T?

b) Is T a one-to-one transformation? Circle one:

Yes No

c) Is T an onto transformation? Circle one:

Yes No

7) A is an $n \times n$ real matrix that has λ as a real eigenvalue. Prove that the (real) eigenspace for λ is a subspace of R^n, as we have done in class. Show all steps! (8 points)

8) Orthogonally diagonalize the real symmetric matrix $A = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$

by giving an orthogonal matrix P and a diagonal matrix D such that $D = P^{-1}AP$ (or, equivalently, $D = P^T AP$). Hint: The eigenvalues of A are 0 and 5; you don't have to show that. (16 points)

YOU MAY USE THE NEXT PAGE! →

8) (cont.)

9) Rewrite $\dfrac{1}{3-2i}\begin{bmatrix} 4+i & i \\ -2 & 0 \end{bmatrix}$ as an equivalent expression of the form $c\begin{bmatrix} z_1 & z_2 \\ z_3 & z_4 \end{bmatrix}$, where c is a real number, and the "z_i"s are complex numbers written in standard form. (8 points)

10) Find $\|\mathbf{v}\|$, the Euclidean norm of $\mathbf{v}$, where $\mathbf{v} = (4+i, 3)$ is a vector in C^2. (4 points)

11) A is an $n \times n$ real matrix. For each statement below, circle "Yes" if it is equivalent to the statement **"A is invertible."** Otherwise, circle "No". You do <u>not</u> have to justify your answers. (10 points total; 2 points each)

a) $\det(A) = 0$. Yes No

b) A is diagonalizable. Yes No

c) A can be written as the product of elementary matrices. Yes No

d) $\operatorname{rank}(A) = n$. Yes No

e) A is similar to I_n, the $n \times n$ identity matrix. Yes No

<u>TRUE or FALSE (12 points; 3 points each)</u>

Circle "True" or "False" (as appropriate) for each statement below.
You do <u>not</u> have to justify your answers.

Remember that you must circle "False" if the statement is "sometimes true, but sometimes false."

Assume that m and n represent positive integers and that all matrices are real.

1) If A and B are $n \times n$ matrices, then $(A+B)(A-B) = A^2 - B^2$.

True False

2) If A is an $m \times n$ matrix, then $A^T A$ is symmetric.

True False

3) A set of 7 vectors in R^5 must span R^5.

True False

4) If A is an $m \times n$ matrix, then the set of all (compatible) vectors $\mathbf{b}$ that make $A\mathbf{x} = \mathbf{b}$ consistent is a subspace of R^m.

True False