Math 254 Name: ________________________

MIDTERM 1

MATH 254 - SUMMER 2002 - KUNIYUKI
CHAPTERS 1, 2, 3

GRADED OUT OF 75 POINTS $\times$ 2 = 150 POINTS TOTAL

Circle your final answers!
Show all work and simplify wherever appropriate, as we have done in class!

1) Use either Gaussian elimination with back-substitution or Gauss-Jordan elimination to solve the following system. You must use matrices, as we have done in class. Write your answer as an ordered triple of the form (x_1, x_2, x_3).

$$\begin{cases} 4x_1 - 4x_2 + 14x_3 = 50 \\ 3x_2 + 12x_3 = 90 \\ x_1 - x_2 + 3x_3 = 9 \end{cases}$$

(15 points)

2) Consider the system

$$\begin{cases} x_1 + 2x_2 - x_4 = 6 \\ x_3 + 3x_4 = 7 \end{cases}$$

(10 points total)

a) Write the solution set of the system in parametric form, as we have done in class.

b) Use part a) to find two particular solutions to the system.
Note: There are infinitely many possible answers.

3) Find the matrix product $A^T B$ where $A = \begin{bmatrix} 2 & 0 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 3 & 2 \\ -1 & 0 & 0 \end{bmatrix}$.

(6 points)

4) Find the inverse of the matrix

$$\begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 4 \\ 3 & 0 & 0 \end{bmatrix}$$

(6 points)

5) Let A be the matrix $\begin{bmatrix} 2 & 0 & 1 \\ -4 & 1 & -3 \\ 0 & 4 & -1 \end{bmatrix}$. An LU-factorization of A is given by $A = LU$,

where $L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix}$.

Use this LU-factorization to solve the following system

$$\begin{cases} 2x_1 \qquad + \ x_3 = -13 \\ -4x_1 + x_2 - 3x_3 = 40 \\ \qquad 4x_2 - \ x_3 = 29 \end{cases}$$

(10 points)

6) Find the determinants. (16 points total)

a) $\begin{vmatrix} 1 & 3 & 4 & 1 \\ 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 3 \\ -3 & -1 & 1 & -1 \end{vmatrix}$ (8 points)

b) $\begin{vmatrix} 4 & 3 & 0 & 5 \\ 0 & 0 & 0 & 2 \\ -4 & -3 & 3 & -4 \\ 0 & 2 & 4 & -1 \end{vmatrix}$ (6 points)

Find the determinant by first reducing the matrix to triangular form.

c) $\begin{vmatrix} 4 & 3 & 4 \\ 1 & 10 & 1 \\ -7 & 5 & -7 \end{vmatrix}$ (2 points)

TRUE or FALSE (12 points; 3 points each)

Circle "True" or "False" (as appropriate) for each statement below.
You do <u>not</u> have to justify your answers.

Remember that you must circle “False” if the statement is “sometimes true, but sometimes false.”

Assume that n represents a positive integer.

a) A homogeneous system of three linear equations in five unknowns (variables) must have infinitely many solutions.

True False

b) If A is an invertible $n \times n$ real matrix, then $A\mathbf{x} = \mathbf{b}$ is a consistent system for any $n \times 1$ real vector $\mathbf{b}$.

True False

c) If A and B are invertible $n \times n$ matrices, then $(AB)^{-1} = A^{-1}B^{-1}$.

True False

d) If A and B are invertible $n \times n$ matrices, then $\det\left(B^{-1}AB\right) = \det(A)$.

True False