Math 254 Name: ________________________

MIDTERM 2

MATH 254 - SUMMER 2002 - KUNIYUKI
CHAPTERS 4, 5

GRADED OUT OF 75 POINTS $\times$ 2 = 150 POINTS TOTAL
Circle your final answers!
Show all work and simplify wherever appropriate, as we have done in class!
A scientific calculator is allowed on this exam.

Assume that the "standard" operations for vector addition and scalar multiplication are being used in all relevant problems.

1) For each set below, circle "Yes" if it is a vector space, or circle "No" if it is not.
   - **Whenever you answer "Yes", you do not have to justify your answer.**
   - **Whenever you answer "No", give a counterexample.**

   (10 points total)

a) The set $\left\{(x,y) \,\middle|\, x \text{ is an integer and } y \text{ is a real number}\right\}$

Yes No

b) The set of all fourth-degree polynomials in x

Yes No

c) Span $\left(\left\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\right\}\right)$, where $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are vectors in R^6

Yes No

2) Let W be the set of all 3×3 real diagonal matrices. Prove that W is a subspace of $M_{3,3}$. You may assume that $M_{3,3}$ is, itself, a vector space without having to prove that. (Completeness and quality are criteria for grading your proof.) (10 points)

3) For each of the following sets of vectors, circle "Linearly independent" or "Linearly dependent" as appropriate. You do not have to justify your answers. (12 points; 3 points each)

a) {(1, 2, 7), (3, 6, 19)}

Linearly independent   Linearly dependent

b) {(1, 7, 8, 3), (0, 0, 0, 0), (4, -2, 3, 10)}

Linearly independent   Linearly dependent

c) $\{1,\ 1+2x,\ 1+2x+3x^2\}$ (Treat these vectors as vectors in P_2.)

Linearly independent   Linearly dependent

d) The set of four vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, 2\mathbf{v}_1 + \mathbf{v}_2\}$, where $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$ are vectors in R^7.

Linearly independent Linearly dependent

4) Does the set of vectors $\{(1,0,4),\ (0,3,0),\ (3,7,12),\ (4,1,16)\}$ span R^3, or not? **Circle** "Spans R^3" or "Does not span R^3" as appropriate, **and justify your answer. Show all work!** (6 points)

Spans R^3 Does not span R^3

5) Let V be the vector space of all 2×2 real symmetric matrices. What is the dimension of V? You do <u>not</u> have to justify your answer. (3 points)

6) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -8 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{bmatrix}$$

By applying elementary row operations (EROs), A can be reduced to the following reduced row-echelon (RRE) form matrix:

$$B = \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(12 points total)

a) What is the rank of A? (2 points)

b) Write a basis for Row(A), the row space of A. (4 points)

c) Write a basis for Col(A), the column space of A. (4 points)

d) What is the nullity of A? (2 points)

7) Let **v** and **w** be two vectors in R^4 such that **v** and **w** are orthogonal, $\| \mathbf{v} \| = 10$, and $\| \mathbf{w} \| = 6$. Find $(\mathbf{v} + 2\mathbf{w}) \bullet (\mathbf{v} - \mathbf{w})$. Your final answer will be a real number. (6 points)

8) The set of vectors below is a basis for a two-dimensional subspace of R^4. Use the Gram-Schmidt orthonormalization process to transform this basis

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 10 \\ 8 \\ 0 \end{bmatrix} \right\}$$

into an orthonormal basis for the same subspace.

Hint: The orthogonal projection of a vector **v** onto a vector **w** is given by $\text{proj}_{\mathbf{w}} \mathbf{v} = \left(\frac{\mathbf{v} \bullet \mathbf{w}}{\mathbf{w} \bullet \mathbf{w}} \right) \mathbf{w}$.

(16 points)

YOU MAY USE THE BACK OF THIS SHEET →