Math 254 Name: ________________________

MIDTERM 3

MATH 254 - SUMMER 2002 - KUNIYUKI
CHAPTERS 6, 7

GRADED OUT OF 75 POINTS × 2 = 150 POINTS TOTAL
Circle your final answers!
Show all work and simplify wherever appropriate, as we have done in class!
A scientific calculator is allowed on this exam.

1) The linear transformation $T: R^2 \to R^3$ is such that $T(1,1) = (-3,4,1)$ and $T(0,1) = (1,5,3)$. Find $T(4,5)$.
Hint: Remember the definition of a linear transformation.
(5 points)

2) The linear transformation $T: R^2 \to R^2$ is such that $T(v_1, v_2) = (2v_1 - v_2\,,\ 2v_1 + 3v_2)$. Find the preimage of $(17,5)$.
(7 points)

3) The linear transformation $T: R^5 \to R^7$ is such that $\dim(\text{Ker}(T)) = 3$.
(11 points total)

a) What is the domain of T?

b) What is $\text{nullity}(T)$?

c) What is $\text{rank}(T)$?

d) True or False: $\text{Range}(T)$ is a subspace of R^7. Circle one:

True False

e) True or False: $\text{Ker}(T)$ is a subspace of R^7. Circle one:

True False

4) $T: R^2 \to R^2$ is a linear transformation such that, relative to the standard basis of R^2, $T(x, y) = T(3x - y,\ x + 4y)$. Another basis for R^2 is given by: $B' = \{(1,3),(2,0)\}$. (13 points total)

a) Find the standard matrix for T. (3 points)

b) Find the matrix for T relative to the basis B'. (10 points)

YOU MAY CONTINUE ON THE NEXT PAGE.

5) Find the eigenvalues of the following matrix. Show all work!

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 7 \end{bmatrix}$$

(6 points)

6) Diagonalize the matrix $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$ by giving matrices P and D such that $D = P^{-1}AP$, where D is diagonal. You do <u>not</u> have to give P^{-1}.
(16 points)

7) Prove that if A is an orthogonal matrix, then $|A|$ is either 1 or -1.
Show all steps! (5 points)

TRUE or FALSE (12 points; 3 points each)

Circle "True" or "False" (as appropriate) for each statement below.
You do not have to justify your answers.

Assume that A is a real matrix in the statements below.

1) If the linear transformation $T: V \rightarrow W$ is an isomorphism, then $\operatorname{Ker}(T)$ must only consist of a "zero" vector.

True False

2) If A is a symmetric matrix, then A must be similar to a diagonal matrix.

True False

3) If A is a square matrix that has 4 as an eigenvalue with algebraic multiplicity 3, then the eigenspace for $\lambda = 4$ must be three-dimensional.

True False

4) If A is a square matrix whose columns form an orthogonal set of nonzero vectors, then A must be an orthogonal matrix.

True False