Math 254 Name: ________________________

FINAL

MATH 254 - SUMMER 2001 - KUNIYUKI

Show all appropriate work (as we have done in class) for full credit!
A scientific calculator is allowed on this quiz.
This final will be scored out of 100 points; the score will then be doubled.
You do not have to rationalize denominators or simplify radicals (e.g., $\sqrt{12} = 2\sqrt{3}$).

1) Find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 4 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

(8 points)

2) Find the determinant of the matrix

$$A = \begin{vmatrix} 0 & 0 & 0 & 2 \\ 3 & 1 & 2 & 7 \\ 6 & -2 & 3 & -1 \\ 5 & 0 & 0 & 4 \end{vmatrix}.$$

(10 points)

3) Let $\mathbf{v}_1 = (1,2)$ and $\mathbf{v}_2 = (3,5)$. Express (-7,-9) as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$. There should be no unknowns in your final expression. (10 points)

4) The vectors **x**, **y**, and **z** are three vectors in R^n for some fixed *n*.
Prove that if **x** is orthogonal to both **y** and **z**, then **x** is orthogonal to any linear combination of **y** and **z**. (10 points)

5) The linear transformation $T: R^5 \rightarrow R^4$ is defined by $T(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{bmatrix} 1 & 0 & 4 & 3 & 0 \\ 0 & 1 & -2 & 5 & 0 \\ 0 & 0 & 0 & 0 & 3 \\ 2 & 0 & 8 & 6 & 0 \end{bmatrix}.$$

Find a basis for the Range of T. (10 points)

6) Orthogonally diagonalize the symmetric matrix

$$A = \begin{bmatrix} 1 & 4 \\ 4 & 1 \end{bmatrix}$$

by giving an orthogonal matrix P and a diagonal matrix D such that $D = P^{-1}AP$ (or, equivalently, $D = P^{T}AP$).

<u>Hint:</u> The eigenvalues of this matrix (A) are -3 and 5.

(There is space on the back of this sheet, so you can continue your work.)

(25 points)

You can continue on the back of this sheet.
Remember that P must be an <u>orthogonal</u> matrix!

6) cont.)

7) A and P are $n \times n$ matrices. If det(A) = 4, and det(P) = 7, what is $\det\left(P^{-1}AP\right)$?
(6 points)

TRUE OR FALSE

For each statement, circle "True" or "False" as appropriate.
You do not have to justify any of your responses.
(21 points; 3 points each)

a) A homogeneous system of linear equations must be consistent.

True False

b) If A, B, and C are $n \times n$ matrices, and if $AB = AC$, then it must be true that $B = C$.

True False

c) All elementary matrices are invertible.

True False

d) The set of rational numbers is a subspace of **R**, the vector space of real numbers.

True False

e) A set of five vectors in R^3 must be linearly dependent.

True False

f) A set of five vectors in R^3 must span R^3.

True False

g) If the linear transformation $T: R^3 \to R^4$ is defined by $T(\mathbf{x}) = A\mathbf{x}$, where

$A = \begin{bmatrix} 2 & 3 & 2 \\ 0 & 1 & -5 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$, then T is a one-to-one transformation.

True False