Math 254 Name: ________________________

QUIZ 1

MATH 254 - SUMMER 2001 - KUNIYUKI
CHAPTERS 1, 2, 3

Show all appropriate work (as we have done in class) for full credit!
A scientific calculator is allowed on this quiz.

1) Solve the system below using Gauss-Jordan elimination. Write the solution set in parametric form. (You have room on the back of this sheet.) (20 points)

$$\begin{cases} 3x_1 - 2x_2 + 6x_3 - 5x_4 = 2 \\ x_2 - 3x_3 = -5 \\ 6x_1 - 4x_2 + 12x_3 - 7x_4 = 16 \end{cases}$$

1) cont.)

2) Find the matrix $\left(X^T X\right)^{-1}$ if X is the matrix $\begin{bmatrix} 1 & -5 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}$. (15 points)

3) If A is an invertible matrix, find an expression for the solution $\mathbf{x}$ to the system $A\mathbf{x} = \mathbf{b}$ and prove that it is unique (as we have done in class). Assume that all sizes are compatible. (Your proof of uniqueness will not require uniqueness of A^{-1}.) (5 points)

4) Let A be the matrix $\begin{bmatrix} 1 & 5 & -4 \\ 0 & 3 & 1 \\ 4 & 14 & -14 \end{bmatrix}$. (20 points total)

a) Find an LU-factorization of A. (10 points)

b) Use a) to solve the following system. (10 points)

$$\begin{cases} x_1 + 5x_2 - 4x_3 = -10 \\ 3x_2 + x_3 = 0 \\ 4x_1 + 14x_2 - 14x_3 = -28 \end{cases}$$

5) Find the determinant below. (20 points)

$$\begin{vmatrix} 1 & 7 & 0 & -2 & 0 \\ 3 & -8 & 0 & 5 & 5 \\ 4 & -3 & 0 & 6 & 0 \\ -1 & 7 & 4 & -3 & 15 \\ 4 & 2 & 0 & 1 & 0 \end{vmatrix}$$

6) If A is a 10×10 matrix, and the determinant of A is 3, find $|2A|$. (3 points)

7) Prove that if A is invertible, $|A^{-1}| = \frac{1}{|A|}$. (5 points)

TRUE or FALSE (12 points; 3 points each)

Circle "True" or "False" (as appropriate) for each statement below.

1) A system of linear equations with more variables than equations can have exactly one solution.

True False

2) A system of linear equations with more equations than variables can have exactly one solution.

True False

3) If a 7×7 matrix has 7 "leading ones" in its reduced row-echelon (RRE) form, it must be the identity matrix I_7.

True False

4) If A and B are same-size square matrices, then $(A - B)^2 = A^2 - 2AB + B^2$.

True False