Math 254 Name: ________________________

QUIZ 2

MATH 254 - SUMMER 2001 - KUNIYUKI
CHAPTER 4

Show all appropriate work (as we have done in class) for full credit!
A scientific calculator is allowed on this quiz.

1) If **v** = (3, 0, -2) and **w** = (-4, 2, 1), find 3**v** - 4**w**. (4 points)

2) (Deleted) (4 points)

3) Let A be a fixed 3×5 matrix. Prove that the set $\left\{ \mathbf{x} \in R^5 : A\mathbf{x} = \mathbf{0} \right\}$ is a subspace of R^5 by proving that it is closed under vector addition and scalar multiplication.
("$\in$" means "is a member of")
(8 points)

4) For each of the following sets, circle "Yes" if it is a vector space, or circle "No" if it is not. Assume that the "standard" operations for vector addition and scalar multiplication are being used in each case. (20 points; 4 points each)

a) The set of all polynomials in x whose degree is exactly three

Yes No

b) The set of all real 4×3 matrices

Yes No

c) The set of standard basis vectors in R^5

Yes No

d) The set of all real 2×2 matrices in upper triangular form

Yes No

e) The set $\{(x, 0, 2x) : x \text{ is a real number}\}$

Yes No

5) For each of the following sets of vectors, circle "Linearly independent" if it is linearly independent, or "Linearly dependent" if it is linearly dependent.
(24 points; 4 points each)

a) {(4, 3), (0, 0)}

Linearly independent Linearly dependent

b) {(2, 5, -4), (4, 10, -8)}

Linearly independent Linearly dependent

c) {(1, 0, 2), (2, 4, 4), (0, 1, 0)}

Linearly independent Linearly dependent

d) {(4, 0, 0), (2, 4, 0), (1, 3, -2)}

Linearly independent Linearly dependent

e) Any set of 10 vectors in R^7.

Linearly independent Linearly dependent

f) A set of three vectors $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ in R^4 with the property that $\mathbf{v_1} - 3\mathbf{v_2} + 2\mathbf{v_3} = \mathbf{0}$.

Linearly independent Linearly dependent

6) Consider the vector space $M_{2,2}$, the set of real 2×2 matrices. (6 points total)

a) Write the standard basis for this vector space. (4 points)

b) What is the dimension of this vector space? (2 points)

7) True or False: A set of 10 vectors that span R^{10} must be a basis for R^{10}.
(4 points)

8) (26 points total)

Consider the matrix

$$A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix}$$

By applying elementary row operations (EROs), A can be reduced to the following reduced row-echelon form matrix:

$$B = \begin{bmatrix} 1 & -3 & 0 & 5 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a) What is the rank of A? (4 points)

b) Write a basis for Row(A), the row space of A. (6 points)

c) Write a basis for Col(A), the column space of A. (6 points)

Reminder:

$$A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -3 & 0 & 5 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

d) Write a basis for $N(A)$, the nullspace of A. (10 points)

9) The rank of a 7×10 matrix is 4. What is the nullity (i.e., the dimension of the nullspace) of the matrix? (4 points)