Math 254 Name: ________________________

QUIZ 3

MATH 254 - SUMMER 2001 - KUNIYUKI
CHAPTERS 5, 6

Show all appropriate work (as we have done in class) for full credit!
A scientific calculator is allowed on this quiz.
You do not have to rationalize denominators or simplify radicals (e.g., $\sqrt{12} = 2\sqrt{3}$).

1) If **v** = (4, 2, -3), find the unit vector in the direction of **v**. In other words, normalize **v**. (5 points)

2) Find the angle between the vectors **v** = (3, 1, 0) and **w** = (2, -3, 5). Indicate whether your final answer is in degrees or in radians. If you are using degrees, round off your answer to the nearest hundredth of a degree; if you are using radians, round off your answer to three decimal places. (15 points)

3) If **v** and **w** are orthogonal vectors, what must be their dot product? (2 points)

4) The set of vectors below is a basis for a two-dimensional subspace of R^3.
Use the Gram-Schmidt orthonormalization process to transform this basis

$$\left\{ \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 14 \\ -7 \end{bmatrix} \right\}$$

into an orthonormal basis for the same subspace. (25 points)

5) The set $\{\mathbf{b_1}, \mathbf{b_2}\}$ is a basis for R^2. Let $T\!:\!R^2 \to R^2$ be a linear transformation such that $T(\mathbf{b_1}) = (3, 5)$ and $T(\mathbf{b_2}) = (-1, 4)$. Find $T(2\mathbf{b_1} + 5\mathbf{b_2})$. (7 points)

6) The linear transformation $T\!:\!R^5 \to R^4$ is defined by $T(\mathbf{x}) = A\mathbf{x}$ for some matrix A. (8 points total)

a) What is the size of A? (4 points)

b) If rank(T) = 3, what is the dimension of Ker(T)? (4 points)

7) The linear transformation T is defined by $T(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{bmatrix} 1 & -2 & 4 & 5 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Assume that the domain of T is R^5 and the codomain is R^4.
(12 points total; 4 points each)

a) What is the dimension of Range(T)?

b) What is nullity(T)?

c) Yes or No: Is T an onto transformation?

8) Prove that if A and B are similar $n \times n$ matrices, then A^2 is similar to B^2.
(6 points)

9) Let the linear transformation $T: R^2 \rightarrow R^2$ be defined by

$$T(x, y) = (x + 3y, 2x - y)$$

(20 points total)

a) Find the standard matrix for *T*. (5 points)

b) Let $B = \{(1,0), (0,1)\}$, the standard basis of R^2.
Let $B' = \{(1,4), (2,3)\}$. Find the matrix for *T* relative to B'.
(15 points)