

**QUIZ 4****MATH 254 - SUMMER 2001 - KUNIYUKI  
CHAPTERS 7, 8**

Show all appropriate work (as we have done in class) for full credit!

A scientific calculator is allowed on this quiz.

You do not have to rationalize denominators or simplify radicals (e.g.,  $\sqrt{12} = 2\sqrt{3}$ ).

- 1) Find the eigenvalues of  $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$ . (20 points)

2) Diagonalize the matrix  $A = \begin{bmatrix} 2 & 2 \\ -1 & 5 \end{bmatrix}$  by giving matrices  $P$  and  $D$  such that  $D = P^{-1}AP$ , where  $D$  is diagonal. You do not have to give  $P^{-1}$ . Hint: The eigenvalues of this matrix are 3 and 4. (30 points)

3) The matrix  $A$  below is an orthogonal matrix. Write  $A^{-1}$ , the inverse matrix. (5 points)

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix}$$

4) Let  $A = \begin{bmatrix} 2i & 3-i \\ 4 & 5i \end{bmatrix}$  and  $B = \begin{bmatrix} 2+i & -i \\ 4i & 3 \end{bmatrix}$ . Find the matrix product  $AB$ . The entries of your final answer must be in standard form. (15 points)

5) Let  $z = 4 - 5i$ . Find  $|z|$ . (5 points)

6) Perform the division and write your final answer in standard form:

$$\frac{3i}{2 - 5i}$$

(10 points)

7) The linear transformation  $T: C^2 \rightarrow C^2$  is given by  $T(\mathbf{v}) = A\mathbf{v}$  where

$$A = \begin{bmatrix} 0 & 4 \\ 4i & 2i \end{bmatrix}$$

(15 points total)

a) Find the image of  $\mathbf{v} = \begin{bmatrix} 4 \\ 3+2i \end{bmatrix}$  under this transformation. (6 points)

b) Find the preimage of  $\mathbf{w} = \begin{bmatrix} 12 \\ 8i \end{bmatrix}$ . (9 points)