CONIC SECTIONS

A video on conic sections may be found here:

https://www.youtube.com/watch?v=HO2zAU3Eppo

Some corrections:

- The definition of **directrix** in the video is not commonly accepted.
- A true cone technically has no "base"; it consists of two **nappes** of infinite surface area.

PARABOLAS: FOCUS AND DIRECTRIX

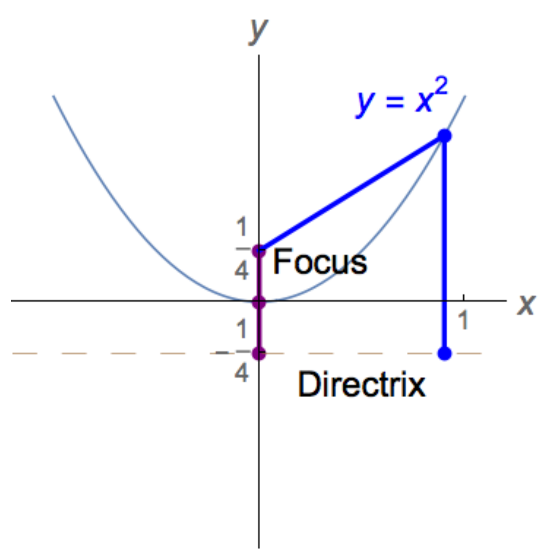
(These comments about parabolas will not be on the Final.)

LEARNING OBJECTIVES

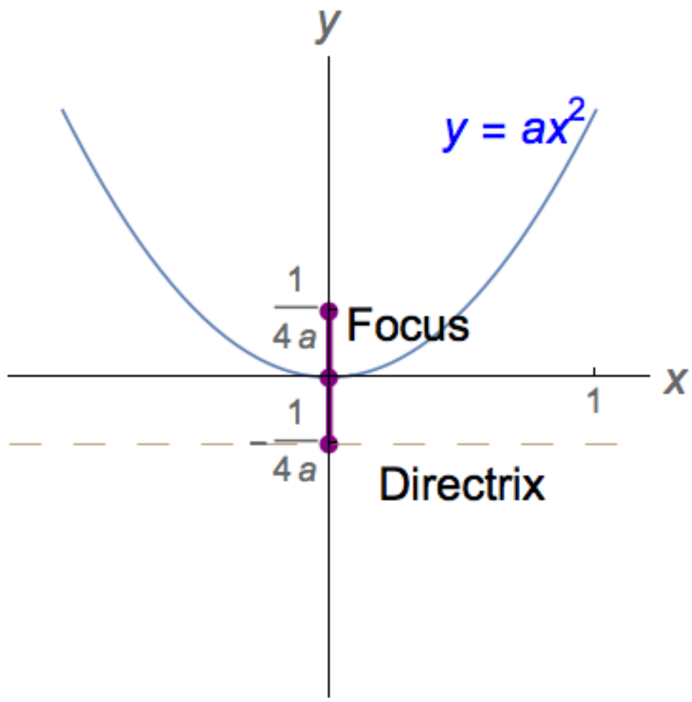
- Know the locus (geometric) definition of a parabola.
- Locate the focus and directrix of a parabola.

PART A: THE LOCUS (GEOMETRIC) DEFINITION OF A PARABOLA

- A **parabola** consists of all points that are **equidistant** between a fixed point (called the focus) and a fixed line (called the directrix).
- The **vertex** of the parabola $y = x^2$, which is at $(0, 0)$, has **distance** $\frac{1}{4}$ from both the **focus** and the **directrix**.



• More generally, the **vertex** of the parabola $y = ax^2$ $(a > 0)$ has **distance** $\frac{1}{4a}$ from both the **focus** and the **directrix**.



• A video on how to **construct a parabola** is here; the **directrix** would be somewhere in the middle of the page, not at the bottom:

https://www.youtube.com/watch?v=M9g7jXrMyeo

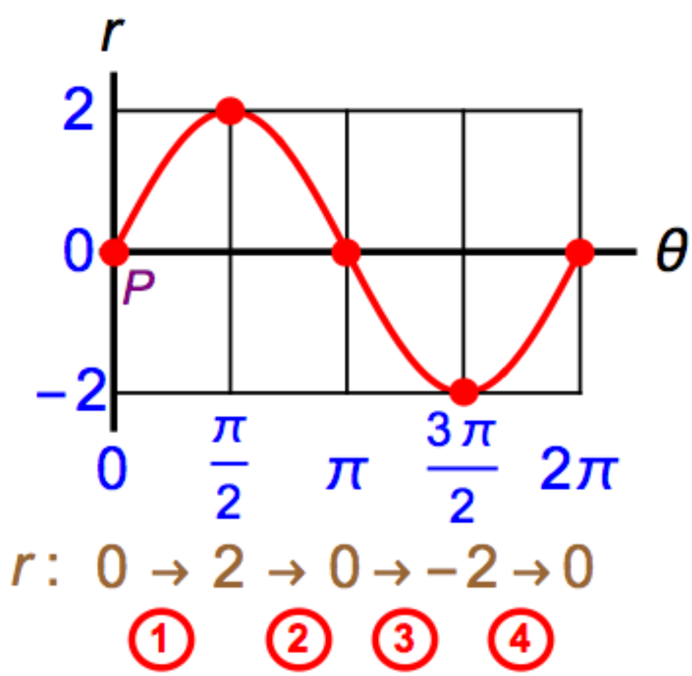
POLAR CURVES: SOME EXAMPLES

Example 1 (Graphing a Circle)

Sketch the graph of the polar equation $r = 2\sin(\theta)$, where r and θ are polar coordinates.

§ Solution

First, graph r against θ as **Cartesian** coordinates. Graph one cycle of $r = 2\sin(\theta)$.



Now, graph r and θ as **polar** coordinates. Draw in Quadrant …

As $\theta:\ 0 \to \frac{\pi}{2}$, $r:\ 0 \to 2$. (Phase 1) I

As $\theta:\ \frac{\pi}{2} \to \pi$, $r:\ 2 \to 0$. (Phase 2) II

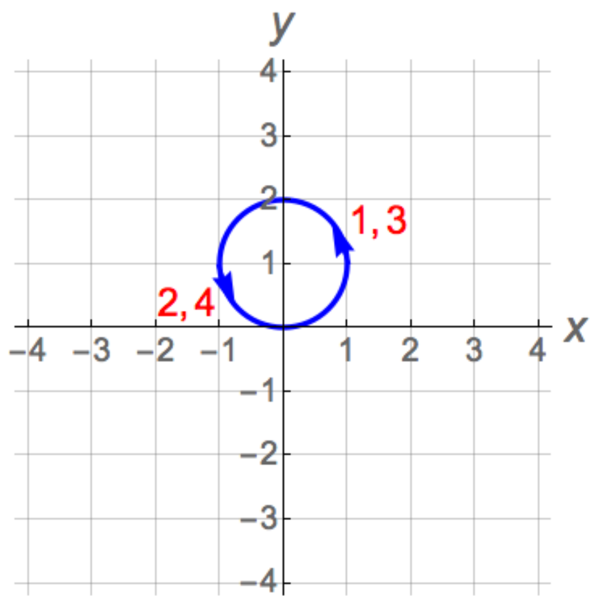
As $\theta:\ \pi \to \frac{3\pi}{2}$, $r:\ 0 \to -2$. (Phase 3) I (not III, because $r \le 0$)

As $\theta:\ \frac{3\pi}{2} \to 2\pi$, $r:\ -2 \to 0$. (Phase 4) II (not IV, because $r \le 0$)

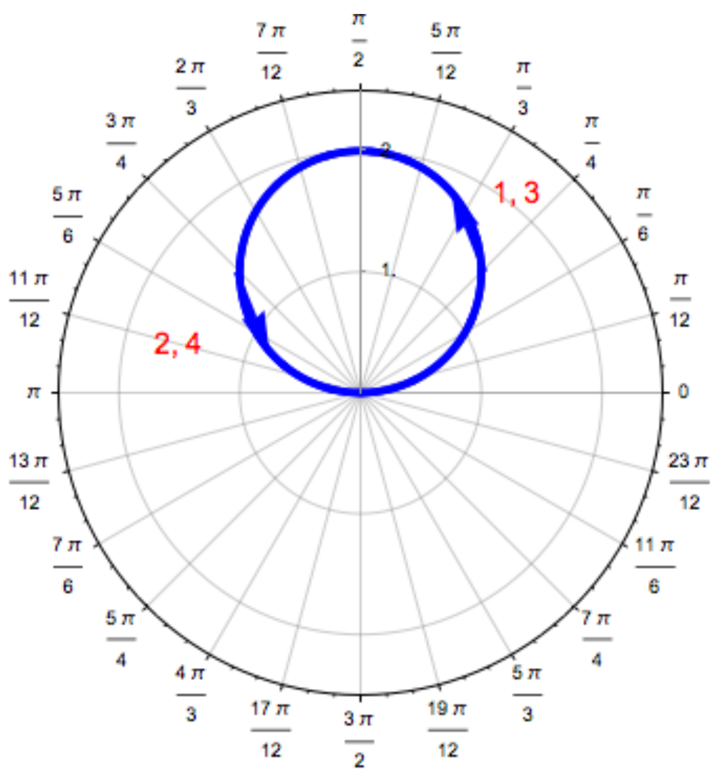
A "**new phase**" should be started whenever:

- θ changes Quadrant.
- r changes direction (from increasing to decreasing, or vice-versa).
- r changes sign.

Here is the polar curve using **Cartesian** graph paper:



Here is the polar curve using **polar** graph paper:



Obtaining the equation of the circle in Cartesian coordinates:

$$r = 2\sin(\theta) \Rightarrow \text{(Multiply both sides by } r.)$$

$$r^2 = 2r\sin(\theta)$$

We need not exclude the case $r = 0$, since 0 is in the range of the $2\sin(\theta)$ function. The **pole (origin)**, which corresponds to $r = 0$, lies on the graph.

$$x^2 + y^2 = 2y$$

$$x^2 + y^2 - 2y = 0$$

$$x^2 + (y^2 - 2y + 1) = 0 + 1 \quad \text{(CTS and balance.)}$$

$$x^2 + (y-1)^2 = 1$$

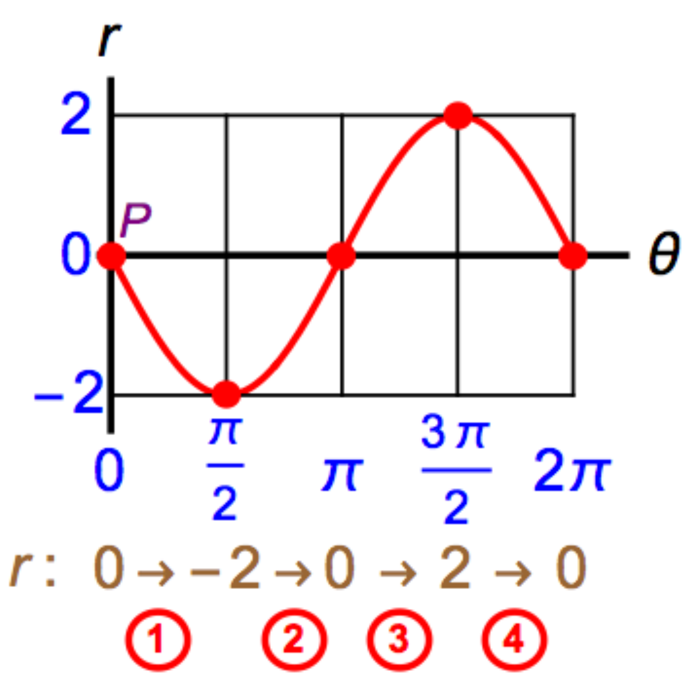
We have a **circle** of radius 1 centered at $(0,1)$. However, this analysis does **not** indicate **orientation**. §

Example 2 (Graphing a Circle)

Sketch the graph of the polar equation $r = -2\sin(\theta)$, where r and θ are polar coordinates.

§ Solution

First, graph r against θ as **Cartesian** coordinates. Graph one cycle of $r = -2\sin(\theta)$.



Now, graph r and θ as **polar** coordinates. Draw in Quadrant …

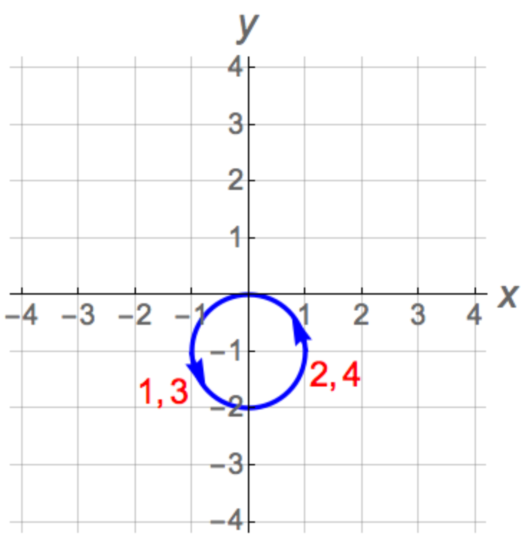
As $\theta: 0 \to \frac{\pi}{2}$, $r: 0 \to -2$. (Phase 1) III (not I, because $r \le 0$)

As $\theta: \frac{\pi}{2} \to \pi$, $r: -2 \to 0$. (Phase 2) IV (not II, because $r \le 0$)

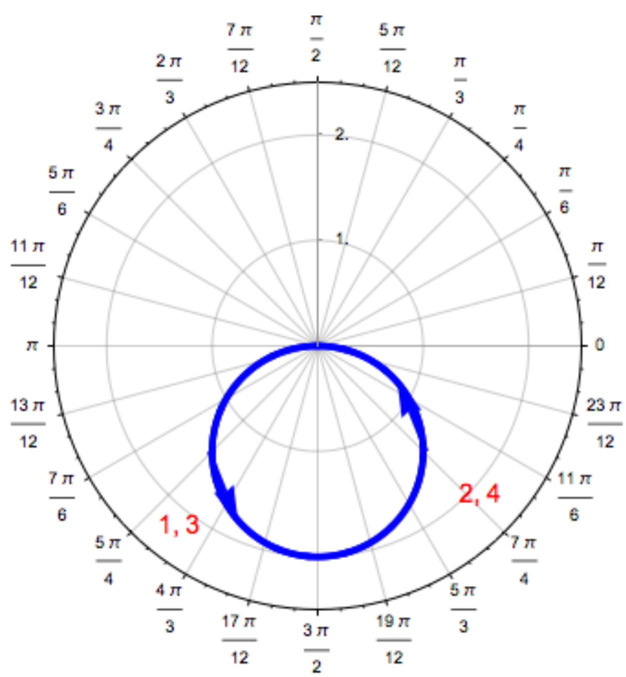
As $\theta: \pi \to \frac{3\pi}{2}$, $r: 0 \to 2$. (Phase 3) III

As $\theta: \frac{3\pi}{2} \to 2\pi$, $r: 2 \to 0$. (Phase 4) IV

Here is the polar curve using **Cartesian** graph paper:



Here is the polar curve using **polar** graph paper:



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