

LESSON 11: EXERCISES

- 1) For an event A , $P(A) = 0.2$. If the event \bar{A} is the **complement** of A , what is $P(\bar{A})$?
- 2) A standard six-sided die is rolled. The event A occurs if and only if the die comes up 1, 2, or 3. When does the event \bar{A} occur?
- 3) A standard six-sided die is rolled. What is the probability that the die comes up a 2 or a 6?
- 4) Two standard six-sided dice are rolled. One die is red; the other is green. The “total” of the two dice is the sum of the numbers on the dice.
 - a) What is the probability of getting a 3 on the green die?
 - b) What is the probability of getting a 5 on the red die and a 6 on the green die?
 - c) What is the probability of getting a 6 on the red die and a 5 on the green die?
 - d) What is the probability that the total is 11?
 - e) What is the probability that the total is 12?
 - f) What is the probability that the total is 2?
 - g) What is the probability that the total is 7?
 - h) What is the probability that the number on the green die is one more than the number on the red die?
- 5) A roulette wheel has 38 equally likely numbers: 18 red numbers, 18 black numbers, and 2 green numbers. You bet that a **red** number will come up on the next spin. The wheel is spun. The probability that you will **win** is $\frac{18}{38}$, which simplifies to $\frac{9}{19}$. What is the probability that you will **lose**? Give two ways of figuring out the answer.

LESSON 12: EXERCISES

- 1) Each registered voter in Fredonia is forbidden from being a member of more than one political party.
- 30% of registered voters in Fredonia are members of the Orange Party; that is, the probability that a randomly selected registered voter in Fredonia is in the Orange Party is 0.3.
 - 40% of registered voters in Fredonia are members of the Purple Party; that is, the probability that a randomly selected registered voter in Fredonia is in the Purple Party is 0.4.

What is the probability that a randomly selected registered voter in Fredonia is a member of the Orange Party or the Purple Party?

- 2) A standard six-sided die is rolled. What is the probability that the die comes up a 6 or a number lower than 4?
- 3) Two standard six-sided dice are rolled. One die is red; the other is green. The “total” of the two dice is the sum of the numbers on the dice. You get “doubles” if and only if the two dice come up the same number.
- a) What is the probability of getting a total of 3 or 5?
 - b) What is the probability of getting doubles or a total of 7?
 - c) What is the probability of getting a 2 or a 4 on the red die or a 3 on the green die?
 - d) What is the probability of getting doubles or a 3 on the green die?
- 4) 35% of adult Fredonians own a cat. 30% of adult Fredonians own a dog. 10% of adult Fredonians own both a cat and a dog. What is the probability that a randomly selected adult Fredonian owns a cat or a dog?
- 5) Fredonia has 10,000 adults. Consider the following two-way frequency (or contingency) table. What is the probability that a randomly selected adult Fredonian owns a cat or a dog? Compare to Exercise 4.

		Owns a Dog?		Total
		Yes	No	
Owns a Cat?	Yes	1000	2500	3500
	No	2000	4500	6500
Total		3000	7000	10,000

LESSON 13: EXERCISES

- 1) A fair coin can only come up heads or tails, each with the same probability. A fair coin is flipped, a standard six-sided die is rolled, and a card is randomly drawn from a standard deck of 52 cards. What is the probability that the coin comes up “heads,” the die comes up a 4, and the card’s suit is “hearts”? That is, find:

$$P(\text{coin-heads and die-4 and card-hearts})$$

Write your answer as an exact fraction.

- 2) Three cards are randomly drawn from a standard 52-card deck **with replacement** (each drawn card is immediately returned to the deck). What is the probability that all three cards are “hearts”? Write your answer as an exact fraction and also round it as a decimal to three significant figures.
- 3) Three cards are randomly drawn from a standard 52-card deck **without replacement** (each drawn card is immediately, permanently removed from the deck). What is the probability that all three cards are “hearts”? Write your answer as an exact fraction and also round it as a decimal to three significant figures. Compare to Exercise 2.
- 4) A valid password to a website must consist of six uppercase English letters. There are 26 uppercase English letters. If you randomly select a valid password, and if a thief randomly selects a valid password, what is the probability that the thief will match your password? Write your answer as an exact fraction.
- 5) In the recent presidential election in Fredonia, the Orange Party candidate received 55.7% of the vote. The Fredonia News Network presents a focus group of ten voters in Fredonia who voted in that election. (The ten voters represented less than 5% of the total number of voters in that election.) The network claims that the ten voters were randomly selected. It turns out that all ten voters voted for the Orange Party candidate.
- a) What is the probability that ten randomly selected voters who voted in that election all voted for the Orange Party candidate? (We may use the Sampling Rule and assume independence.) Write your answer as a decimal rounded off to three significant figures.
 - b) What do you think about the network’s claim that the voters were randomly selected?

LESSON 14: EXERCISES

In this Lesson, if an answer is a probability, then write it as an exact fraction without decimal points.

- 1) Consider a standard deck of 52 cards.
 - a) How many of the cards are red?
 - b) How many of the cards are “hearts”?
 - c) What is the probability that a randomly selected card has the suit “hearts”?
 - d) A friend randomly draws a card from the deck and says that it is a red card. What is the **conditional probability** that the card has the suit “hearts,” **given that** it is red?

- 2) A friend rolls a standard six-sided die and says that it comes up an odd number.
 - a) What is the **conditional probability** that the die came up a 5, **given that** it came up odd?
 - b) What is the **conditional probability** that the die came up a 1 or a 5, **given that** it came up odd?

- 3) We randomly select an adult in Fredonia. The probability that the adult has a dog is 0.30. The probability that the adult owns both a cat and a dog is 0.10. We then find out that the adult owns a dog. Find the **conditional probability** that the adult owns a cat, **given that** the adult owns a dog.

- 4) Fredonia has 10,000 adults. Consider the following two-way frequency (or contingency) table. What is the **conditional probability** that a randomly selected Fredonian adult owns a cat, **given that** the adult owns a dog? Compare to Exercise 3.

		Owens a Dog?		Total
		Yes	No	
Owens a Cat?	Yes	1000	2500	3500
	No	2000	4500	6500
Total		3000	7000	10,000

- 5) Consider the two-way frequency (or contingency) table from Exercise 4. What is the **conditional probability** that a randomly selected Fredonian adult owns a dog, **given that** the adult owns a cat? Compare to Exercise 4; are the probabilities equal?

- 6) For two events A and B , does $P(B|A)$ have to be close to $P(A|B)$?

LESSON 15: EXERCISES

- 1) We can have a probability distribution for a qualitative random variable X . Let X be the suit of a card drawn randomly from a standard deck of 52 cards. Fill out the distribution table below. Hint: The distribution is uniform.

Value (x)	Probability $P(x)$
Clubs	
Diamonds	
Hearts	
Spades	

- 2) Five students take a statistics final. Let X = the number of students who will fail the final. The professor gives the following **incomplete** probability distribution for X . Find the value of a .

Value (x)	Probability $P(x)$
0	0.450
1	0.200
2	0.160
3	0.100
4	0.050
5	a

- 3) Refer to Exercise 2 above. For each of the following, **rewrite** using an inequality, **rewrite** as a sum of probabilities, and **calculate** the indicated probability.
- a) $P(\text{at least } 1)$; also find a faster way of calculating this.
 - b) $P(\text{at most } 4)$; also find a faster way of calculating this.
 - c) $P(\text{more than } 2)$
 - d) $P(\text{at least } 2)$
 - e) $P(\text{fewer than } 3)$
 - f) $P(\text{at most } 3)$; also find a faster way of calculating this.

LESSON 16: EXERCISES

- 1) A “Deal or No Deal” contestant has a case. Let X = the amount of money in the case. X has a **uniform** distribution on the four remaining dollar amounts left in play:

Value (x)	Probability $P(x)$
\$10,000	1/4
\$25,000	1/4
\$75,000	1/4
\$1,000,000	1/4

Find $E(X)$ and **interpret** it.

- 2) Refer to Lesson 15, Exercise 2 on the statistics final. Assume that the statistics professor always has five students taking the final every term. Let X = the number of students who will fail the final. Find $E(X)$ and **interpret** it; assume that the professor’s distribution is correct.
- 3) After you and your family decide to purchase a particular life insurance policy on you, you and your family first pay \$1000 for the year. If you die during the year, your family gets \$100,000. If you do not die during the year, your family gets nothing (beyond your presence, at least). Let X = you and your family’s net monetary gain as a result of the decision to purchase the policy. Based on research, you assume that your probability of dying during the year is 0.004. Assuming this is correct, find $E(X)$ and **interpret** it. Fill out the table:

Outcome for the year	Value (x)	Probability $P(x)$
You live.		
You die.		

LESSON 17: EXERCISES

- 1) For an assignment, a student must interview some group of three professors: one English professor, one Math professor, and one History professor at a school. The school has seven English professors, four Math professors, and two History professors. How many possible groups of three professors are there for the student to complete the assignment?
- 2) In computer science, a bit can be a “0” or a “1.” A byte consists of a sequence of eight bits. How many possible bytes are there?
- 3) You have five shirts for work, and you never reuse a shirt during a five-day work week. In one five-day work week, how many ways are there for you to select shirts throughout the week (that is, for Monday, Tuesday, Wednesday, Thursday, and Friday)?
- 4) There are four singers on a competition show. How many ways are there for a person to list the singers from best to worst (assuming no ties)? Hint: The answer is not “one.”
- 5) There are eight questions on an English exam. You may choose three of the eight questions to write on. How many ways can you choose the three questions, assuming order does not matter?
- 6) There are ten questions on a math test. You answer all of them. No partial credit is given; each answer is either correct or incorrect. How many ways are there for you to get four questions correct on the exam? (For example, one “way” is to get the first four questions correct and the others incorrect.) Assume that the order in which you answer the questions does not matter.

LESSON 18: EXERCISES

1) Refer to Example 2. 70% of American adults like Senator Smith.

That is, $P(\text{a random American adult likes Senator Smith}) = 0.7$.

Six American adults are randomly sampled to form a focus group.

Find the indicated probabilities regarding the people in the focus group who like Senator Smith. Hint: The table for $\text{Bin}(n = 6, p = 0.7)$ is below.

Value (x)	Probability $P(x)$
0	0.001
1	0.010
2	0.060
3	0.185
4	0.324
5	0.303
6	0.118

- a) $P(\text{more than } 4)$
 - b) $P(\text{at least } 5)$
 - c) $P(\text{at most } 4)$
 - d) $P(\text{fewer than } 4)$
- 2) $\text{Bin}(n = 6, p = 0.7)$ is the distribution that described the random variable above in Exercise 1, where we count the number of people in the focus group who like Senator Smith. Similarly, find the distribution that best describes each of the following random variables.
- a) We answer all the questions on a true-false test with fifty questions. We guess randomly on all questions. The random variable is the number of questions you get correct.
 - b) We answer all the questions on a multiple-choice test with thirty questions. Each question has three possible options: “A,” “B,” or “C,” only one of which is correct. We guess randomly on all questions. The random variable is the number of questions you get correct.
 - c) 1% of all adults in Fredonia have cooties. Twenty adult Fredonians are randomly selected. The random variable is the number of selected adult Fredonians who have cooties.