LESSON 19: EXERCISES

1) If $X \sim \text{Uniform}[0, 1]$, find $P(0.1 < X < 0.4)$.

2) If $X \sim \text{Uniform}[0, 1]$, find $P(0.8 < X < 1)$.

3) If $X \sim \text{Uniform}[5, 15]$, find $P(12 < X < 14)$.

4) If $X \sim \text{Uniform}[100, 300]$, find $P(150 < X < 200)$.

LESSON 20: EXERCISES

1) Software tells you that $P(Z < -0.48) \approx 0.3156$.
   - a) Sketch a figure clearly showing this fact. Shade in the relevant region.
   - b) Find $P(Z > -0.48)$. Explain why and sketch a figure clearly showing this. Shade in the relevant region.
   - c) Find $P(Z > 0.48)$. Explain why and sketch a figure clearly showing this. Shade in the relevant region.

2) Software tells you that $P(Z < 2) \approx 0.9772$.
   - a) Sketch a figure clearly showing this fact. Shade in the relevant region.
   - b) Find $P(Z > 2)$. Explain why and sketch a figure clearly showing this. Shade in the relevant region.
   - c) Find $P(Z > -2)$. Explain why and sketch a figure clearly showing this. Shade in the relevant region.

3) Software tells you that $P(Z < -2) \approx 0.0228$ and $P(Z < 2) \approx 0.9772$. Find $P(-2 < Z < 2)$. Sketch a figure clearly showing this. Shade in the relevant region.

4) Software tells you that the $20^{th}$ percentile of the $Z$ distribution is about $-0.84$. Sketch a figure clearly showing this. Shade in the relevant region. Write the corresponding probability statement.
LESSON 21: EXERCISES

1) Let $X$ be the height of a random American man. Then, $X \sim N(\mu = 70 \text{ inches}, \sigma = 4 \text{ inches})$. 
(Source: http://www.usablestats.com/lessons/normal)

Use these hints regarding the $Z$ distribution:

- a) Find $P(X < 60 \text{ inches})$, which is the same as $P(X < 5 \text{ feet})$. First write the corresponding probability expression for $Z$. Show work by using the Formula for $z$ Scores.

- b) Find $P(X > 60 \text{ inches})$, which is the same as $P(X > 5 \text{ feet})$. First write the corresponding probability expression for $Z$.

- c) Find $P(60 \text{ inches} < X < 72 \text{ inches})$, which is the same as $P(5 \text{ feet} < X < 6 \text{ feet})$. First write the corresponding probability expression for $Z$. Use the Formula for $z$ Scores when showing work.

- d) Find the 85th percentile of the distribution of heights of American men. Write the corresponding probability statement for $X$ and interpret it. Hint: The 85th percentile of the $Z$ distribution is about 1.04.
LESSON 22: EXERCISES

1) A machine rolls 100 standard six-sided dice. The distribution for $X$, the result on one die, has:

mean, $\mu = 3.5$

SD, $\sigma \approx 1.7078$

- a) Find the approximate sampling distribution for $\bar{X}$.

- b) Find the probability that the average of the 100 dice ($\bar{X}$) will be between 3.0 and 4.0. That is, find $P(3.0 < \bar{X} < 4.0)$. First write the corresponding probability expression for $Z$. Show work by using the Formula for $z$ Scores for Sample Means. Also compare the result to the one from Example 3.

Use these hints regarding the $Z$ distribution:

2) A large lecture class takes a test. The scores are approximately normally distributed with mean 50 points and standard deviation 10 points. (Assume that partial credit is given so that scores such as 70.1 points are possible.)

Use these hints regarding the $Z$ distribution:
• a) Find the probability that a random student who took the test scored higher than 60 points. First write the corresponding probability expression for \( Z \). Show work by using the Formula for \( z \) Scores.

• b) Four students who took the test are randomly selected. Find the probability that the average of their scores \( \bar{X} \) was higher than 60 points. That is, find \( P(\bar{X} > 60 \text{ points}) \). First find the approximate sampling distribution for \( \bar{X} \). Also write the corresponding probability expression for \( Z \). Show work by using the Formula for \( z \) Scores for Sample Means. Compare the result to the one from a).

**LESSON 23: EXERCISES**

1) A student answers all the questions on a multiple-choice test with 60 questions. Each question has three possible options: “A,” “B,” or “C,” only one of which is correct. The student guesses randomly on all questions. The random variable \( X \) is the number of questions the student gets correct. Approximate the probability that the student gets at least 30 questions correct, \( P(X \geq 30) \), by following these steps:

• a) Describe the distribution of \( X \).

• b) Verify that a normal approximation to the distribution of \( X \) would be appropriate.

• c) Describe the normal distribution that can be used to approximate the distribution of \( X \).

• d) Apply continuity correction(s) and rewrite \( P(X \geq 30) \) in terms of \( X_c \).

• e) Find the \( z \) score for the boundary value of \( X_c \) using the Formula for \( z \) Scores.

• f) Write the corresponding probability expression for \( Z \).

• g) Approximate \( P(X \geq 30) \). Use these hints regarding the \( Z \) distribution:
2) Repeat Exercise 1), parts d) through g), except approximate $P(15 < X < 20)$.

• d) **Apply continuity corrections** and rewrite $P(15 < X < 20)$ in terms of $X_c$.

• e) Find the $z$ scores for the boundary values of $X_c$ using the Formula for $z$ Scores.

• f) Write the corresponding **probability expression for $Z$**.

• g) Approximate $P(15 < X < 20)$. Use these hints regarding the $Z$ distribution: