LESSON 24: EXERCISES

1) 100 adult Fredonian women are randomly sampled, and they have their heights measured. $\bar{x} = 63$ inches, $s = 6$ inches, and $s^2 = 36$ square inches.

   • a) What is a point estimate for the **population mean** of heights of adult Fredonian women?

   • b) What is a point estimate for the **population standard deviation** of heights of adult Fredonian women?

   • c) What is a point estimate for the **population variance** of heights of adult Fredonian women?

2) 2000 registered American voters are randomly sampled and interviewed. 1100 of them approve of Senator Smith.

   • a) Find $\hat{p}$, the **sample proportion** of the interviewed voters who approve of Senator Smith.

   • b) What is a point estimate for the **population proportion** of registered American voters who approve of Senator Smith?

3) Gamblers play a new game at a casino. After 5000 rounds of play, the gamblers have won 2000 of them.

   • a) Find $\hat{p}$, the **sample proportion** of gambler wins among the 5000 rounds of play.

   • b) What is a point estimate for the **probability** that a gambler wins the game in one round of play?

LESSON 25: EXERCISES

1) A **fair coin** may come up heads 49 times in 100 flips. What kind of error does this represent?

2) Let μ be the population mean height of adult Fredonian men. Based on a random sample of adult Fredonian men, we obtain $(65 \text{ inches}, 71 \text{ inches})$ as a 95% confidence interval (CI) for μ.

    - a) What is the **lower limit** of the CI?
    - b) What is the **upper limit** of the CI?
    - c) What is the **sample mean** $\bar{x}$?
    - d) What is the **margin of error** *E* for the CI? Show three different ways of getting *E*.
    - e) **Write the CI** in terms of the values of $\bar{x}$ and *E*.
    - f) **Interpret** the CI.

3) A lecture class takes an exam. We want an interval estimate for μ, the **population mean** of exam scores in the class. A random **sample** of five exams is selected and graded. The sample mean $\bar{x}$ is 65 points. The margin of error *E* for a 90% confidence interval (CI) for μ is 7 points.

    - a) What is the **lower limit** of the CI?
    - b) What is the **upper limit** of the CI?
    - c) **Write the CI** in terms of the values of $\bar{x}$ and *E*.
    - d) **Interpret** the CI.
    - e) Would a **95% CI** be wider or smaller than the 90% CI for μ?

4) What is α for a 90% CI?

5) What is α for an 80% CI?

6) What is α for a 99% CI?

LESSON 26: EXERCISES

1) Consider the standard normal (*z*) distribution.

   • a) What is the **mean**?

   • b) What is the **standard deviation**?

   • c) Yes or No: Is the distribution **symmetric** about its mean?

2) Consider any of the *t* distributions.

   • a) What is the **mean**?

   • b) Is the **standard deviation** equal to 1, less than 1, or greater than 1?

   • c) Yes or No: Is the distribution **symmetric** about its mean?

   • d) As the number of degrees of freedom (df) increases, what distribution will the *t* distributions approach?

3) Consider any of the χ^2 distributions.

   • a) Yes or No: Is the **mean** equal to 0?

   • b) Yes or No: Is the distribution **symmetric** about its mean?

   • c) As the number of degrees of freedom (df) increases, what kind of distribution will the χ^2 distributions approach?

4) In most basic applications, how many degrees of freedom (df) do we use for *t* and χ^2 distributions if the sample size is …

   • a) $n = 10$

   • b) $n = 25$

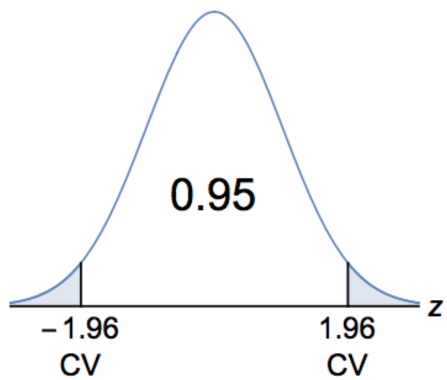
LESSON 27: EXERCISES

None, but familiarize yourself with the terminology and concepts.

LESSON 28: EXERCISES

1) A standardized high school test was designed to have mean 1000 points and population SD 100 points. New teaching methods were then used in high schools. We take a random sample of 50 test results this year, and the sample mean is 970 points. Assume that the population SD is 100 points. You will find a 95% confidence interval (CI) for the population mean of test scores this year.

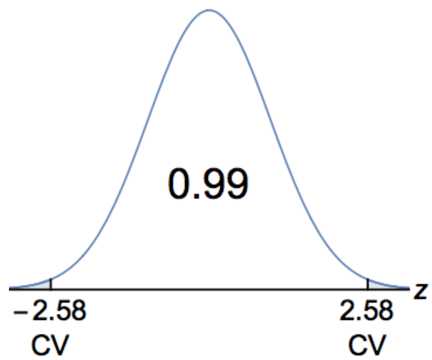
   Use these hints about the *z* distribution:



- a) Why do the methods of this Lesson apply to this problem?
- b) **Write the 95% CI** in the form $\mu = \bar{x} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula.
- c) **Write the 95% CI** in the form $(\text{lower limit, upper limit})$.
- d) **Interpret** the CI.
- e) Based on this analysis, is there sufficient evidence against the claim that the population mean of test scores this year is 1000 points?

2) We will repeat Exercise 1, except that you will find a 99% confidence interval (CI) for the population mean of test scores this year.

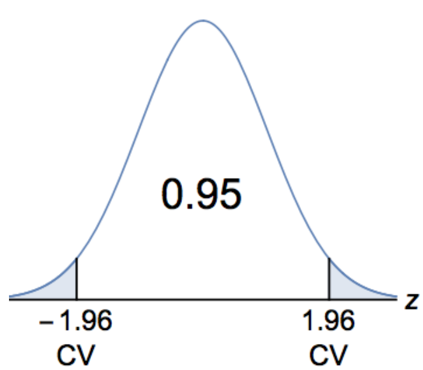
Use these hints about the z distribution:



- a) (Skip this; we did this in Exercise 1.)
- b) **Write the 99% CI** in the form $\mu = \bar{x} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula.
- c) **Write the 99% CI** in the form $\left(\text{lower limit, upper limit}\right)$.
- d) **Interpret** the CI.
- e) Based on this analysis, is there sufficient evidence against the claim that the population mean of test scores this year is 1000 points?

3) The organization that creates the standardized test in Exercises 1 and 2 would like to know the population mean of test scores this year. Assume that the population SD is 100 points. **Find the required sample size** n that would give us a margin of error of 5 points for a 95% confidence interval (CI) for the population mean. Clearly show how this is obtained by plugging into an appropriate formula. **Interpret** your result, as in the notes.

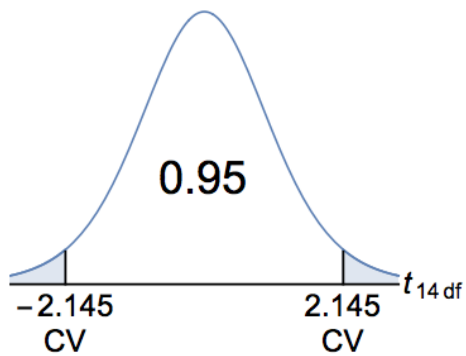
Use these hints about the z distribution:



LESSON 29: EXERCISES

1) A random sample of 15 employees at a large company are surveyed about their incomes. Assume that incomes at the company are approximately normally distributed. The sample mean income is \$60,000, and the sample SD is \$25,000. You will find a 95% confidence interval (CI) for the population mean income at the company.

   Use these hints about the *t* distribution on 14 degrees of freedom (df):



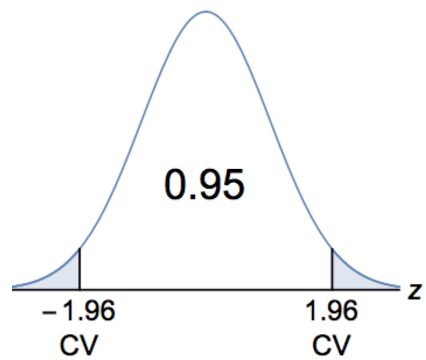
- a) Why do the methods of this Lesson apply to this problem?
- b) **Write the 95% CI** in the form $\mu = \bar{x} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula.
- c) **Write the 95% CI** in the form $(\text{lower limit, upper limit})$.
- d) **Interpret** the CI.

LESSON 30: EXERCISES

1) A magician's coin is flipped 200 times. It comes up heads 110 times. You will find a 95% confidence interval (CI) for the probability that the coin comes up heads on a flip.

   Round off values of $\hat{p}$, $\hat{q}$, and E to three decimal places.

   Use these hints about the z distribution:



- a) Find the sample proportion of heads, $\hat{p}$.
- b) Find the sample proportion of tails, $\hat{q}$.
- c) **Verify** that normal approximations are appropriate in this problem.
- d) **Write the 95% CI** in the form $p = \hat{p} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula.
- e) **Write the 95% CI** in the form $\left(\text{lower limit, upper limit}\right)$.
- f) **Interpret** the CI.
- g) Based on this analysis, is there sufficient evidence against the claim that the coin is fair?
- h) We want to know the probability that the coin comes up heads on a flip. **Find the required sample size** n that would give us a margin of error of 0.05 for a 95% confidence interval (CI) for this probability. Clearly show how this is obtained by plugging into an appropriate formula (the one giving us a conservative estimate).
- i) Is your answer to h) more than, or less than, 200 [flips]? Compare the margin of error in h) with the one from d).

LESSON 31: EXERCISES

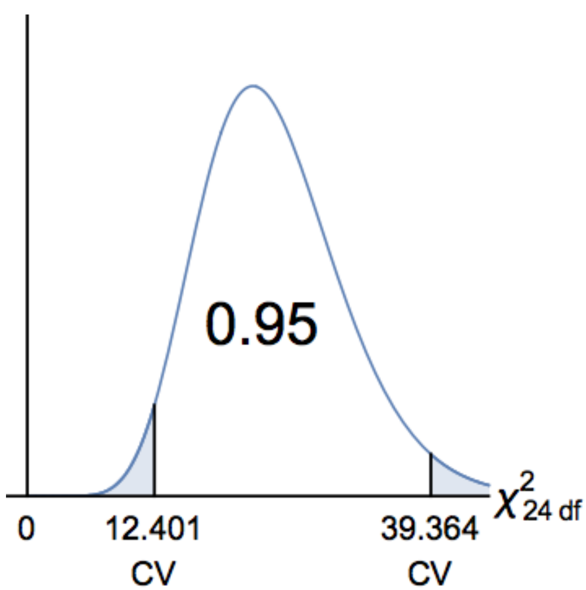
1) A wrench produced by a company is supposed to have size 1 inch. The wrench sizes produced by the company are approximately **normally distributed**. We randomly select 25 wrenches. The sample variance is 0.0025 square inches.

   • a) We will find confidence intervals (CIs) for the population variance and SD of the wrench sizes produced by the company. Why do the methods of this Lesson apply to this problem?

   • b) **Find** a 95% confidence interval (CI) for the population variance of the wrench sizes produced by the company. Clearly show how the limits are obtained by plugging into an appropriate formula. **Write** the CI in either the form $\text{lower limit} < \sigma^2 < \text{upper limit}$ or $(\text{lower limit, upper limit})$. **Interpret** the CI.

   • c) **Also find** a 95% confidence interval (CI) for the population SD of the wrench sizes produced by the company. Clearly show how the limits are obtained by plugging into an appropriate formula. **Write** the CI in either the form $\text{lower limit} < \sigma < \text{upper limit}$ or $(\text{lower limit, upper limit})$. **Interpret** the CI.

   Use these hints about the χ^2 distribution on 24 degrees of freedom (df):



LESSON 32: EXERCISES

1) In a hypothesis test, which do we sometimes accept: H_0 or H_1?

2) A magician says that "4" is the luckiest number. The magician gives you a die. H_0 states that the die is a standard die (with a uniform distribution for its faces). H_1 states that the die is **not** a standard die. You roll the die twice, and it comes up a "4" both times. The probability that a standard die rolled twice comes up a "4" both times by chance is about 0.028. Let's take this as our *P*-value.

   • a) If the significance level $\alpha = 0.05$, would we reject H_0, based on this *P*-value?

   • b) If the significance level $\alpha = 0.01$, would we reject H_0, based on this *P*-value?

3) Refer to Examples 3 and 5. A magician's coin will be flipped 100 times. H_0 states that the coin is **fair**. H_1 states that the coin is **not fair**. Let's say we observe 62 heads among the 100 flips. Let *X* = the number of heads in 100 flips of a fair coin. Use the significance level: $\alpha = 0.05$.

   • a) Using a one-tailed *P*-value analysis,

$$\begin{aligned}\text{One-tailed } P\text{-value} &= P(X \geq 62)\\ &= P(62) + P(63) + P(64) + \ldots + P(100)\\ &\approx 0.0105, \text{ or } 1.05\%\end{aligned}$$

   Do we decide to **reject** or **not reject** H_0? What do we **conclude** about the assumption that the coin is fair?

   • b) Using a two-tailed *P*-value analysis,

$$\begin{aligned}\text{Two-tailed } P\text{-value} &= P(X \geq 62 \ \text{ or } \ X \leq 38)\\ &\approx 0.0210, \text{ or } 2.10\%\end{aligned}$$

   Do we decide to **reject** or **not reject** H_0? What do we **conclude** about the assumption that the coin is fair?

   • c) A 95% confidence interval (CI) for *p*, the probability of heads, is: $(0.525,\ 0.715)$. Do we **reject** or **not reject** H_0 in a two-tailed hypothesis test? What do we **conclude** about the assumption that the coin is fair?

4) Repeat Exercise 3), but use the significance level: $\alpha = 0.01$.

A magician's coin will be flipped 100 times. H_0 states that the coin is **fair**. H_1 states that the coin is **not fair**. Let's say we observe 62 heads among the 100 flips.
Let X = the number of heads in 100 flips of a fair coin.
Use the significance level: $\alpha = 0.01$.

- a) Using a one-tailed P-value analysis,

$$\begin{aligned}\text{One-tailed } P\text{-value} &= P(X \geq 62) \\ &= P(62) + P(63) + P(64) + \ldots + P(100) \\ &\approx 0.0105, \text{ or } 1.05\%\end{aligned}$$

  Do we decide to **reject** or **not reject** H_0? What do we **conclude** about the assumption that the coin is fair?

- b) Using a two-tailed P-value analysis,

$$\begin{aligned}\text{Two-tailed } P\text{-value} &= P(X \geq 62 \text{ or } X \leq 38) \\ &\approx 0.0210, \text{ or } 2.10\%\end{aligned}$$

  Do we decide to **reject** or **not reject** H_0? What do we **conclude** about the assumption that the coin is fair?

- c) A 99% confidence interval (CI) for p, the probability of heads, is: $(0.495,\ 0.745)$. Do we decide to **reject** or **not reject** H_0? What do we **conclude** about the assumption that the coin is fair?