

LESSON 24: EXERCISES

- 1) 100 adult Fredonian women are randomly sampled, and they have their heights measured. $\bar{x} = 63$ inches, $s = 6$ inches, and $s^2 = 36$ square inches.
- a) What is a point estimate for the **population mean** of heights of adult Fredonian women?
 - b) What is a point estimate for the **population standard deviation** of heights of adult Fredonian women?
 - c) What is a point estimate for the **population variance** of heights of adult Fredonian women?
- 2) 2000 registered American voters are randomly sampled and interviewed. 1100 of them approve of Senator Smith.
- a) Find \hat{p} , the **sample proportion** of the interviewed voters who approve of Senator Smith.
 - b) What is a point estimate for the **population proportion** of registered American voters who approve of Senator Smith?
- 3) Gamblers play a new game at a casino. After 5000 rounds of play, the gamblers have won 2000 of them.
- a) Find \hat{p} , the **sample proportion** of gambler wins among the 5000 rounds of play.
 - b) What is a point estimate for the **probability** that a gambler wins the game in one round of play?

LESSON 25: EXERCISES

- 1) A **fair coin** may come up heads 49 times in 100 flips. What kind of error does this represent?
- 2) Let μ be the population mean height of adult Fredonian men. Based on a random sample of adult Fredonian men, we obtain (65 inches, 71 inches) as a 95% confidence interval (CI) for μ .
- a) What is the **lower limit** of the CI?
 - b) What is the **upper limit** of the CI?
 - c) What is the **sample mean** \bar{x} ?
 - d) What is the **margin of error** E for the CI? Show three different ways of getting E .
 - e) **Write the CI** in terms of the values of \bar{x} and E .
 - f) **Interpret** the CI.
- 3) A lecture class takes an exam. We want an interval estimate for μ , the **population mean** of exam scores in the class. A random **sample** of five exams is selected and graded. The sample mean \bar{x} is 65 points. The margin of error E for a 90% confidence interval (CI) for μ is 7 points.
- a) What is the **lower limit** of the CI?
 - b) What is the **upper limit** of the CI?
 - c) **Write the CI** in terms of the values of \bar{x} and E .
 - d) **Interpret** the CI.
 - e) Would a **95% CI** be wider or smaller than the 90% CI for μ ?
- 4) What is α for a 90% CI?
- 5) What is α for an 80% CI?
- 6) What is α for a 99% CI?

LESSON 26: EXERCISES

- 1) Consider the standard normal (z) distribution.
 - a) What is the **mean**?
 - b) What is the **standard deviation**?
 - c) Yes or No: Is the distribution **symmetric** about its mean?
- 2) Consider any of the t distributions.
 - a) What is the **mean**?
 - b) Is the **standard deviation** equal to 1, less than 1, or greater than 1?
 - c) Yes or No: Is the distribution **symmetric** about its mean?
 - d) As the number of degrees of freedom (df) increases, what distribution will the t distributions approach?
- 3) Consider any of the χ^2 distributions.
 - a) Yes or No: Is the **mean** equal to 0?
 - b) Yes or No: Is the distribution **symmetric** about its mean?
 - c) As the number of degrees of freedom (df) increases, what kind of distribution will the χ^2 distributions approach?
- 4) In most basic applications, how many degrees of freedom (df) do we use for t and χ^2 distributions if the sample size is ...
 - a) $n = 10$
 - b) $n = 25$

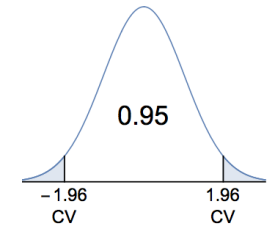
LESSON 27: EXERCISES

None, but familiarize yourself with the terminology and concepts.

LESSON 28: EXERCISES

- 1) A standardized high school test was designed to have mean 1000 points and population SD 100 points. New teaching methods were then used in high schools. We take a random sample of 50 test results this year, and the sample mean is 970 points. Assume that the population SD is 100 points. You will find a 95% confidence interval (CI) for the population mean of test scores this year.

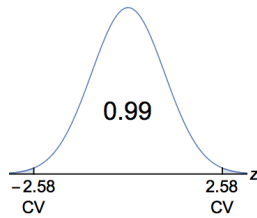
Use these hints about the z distribution:



- a) Why do the methods of this Lesson apply to this problem?
- b) **Write the 95% CI** in the form $\mu = \bar{x} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula.
- c) **Write the 95% CI** in the form (lower limit, upper limit).
- d) **Interpret** the CI.
- e) Based on this analysis, is there sufficient evidence against the claim that the population mean of test scores this year is 1000 points?

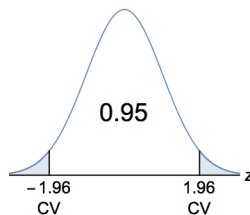
- 2) We will repeat Exercise 1, except that you will find a 99% confidence interval (CI) for the population mean of test scores this year.

Use these hints about the z distribution:



- a) (Skip this; we did this in Exercise 1.)
 - b) **Write the 99% CI** in the form $\mu = \bar{x} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula.
 - c) **Write the 99% CI** in the form (lower limit, upper limit).
 - d) **Interpret** the CI.
 - e) Based on this analysis, is there sufficient evidence against the claim that the population mean of test scores this year is 1000 points?
- 3) The organization that creates the standardized test in Exercises 1 and 2 would like to know the population mean of test scores this year. Assume that the population SD is 100 points. **Find the required sample size n** that would give us a margin of error of 5 points for a 95% confidence interval (CI) for the population mean. Clearly show how this is obtained by plugging into an appropriate formula. **Interpret** your result, as in the notes.

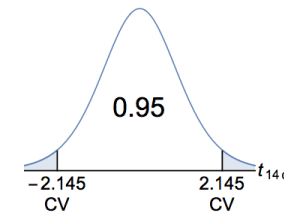
Use these hints about the z distribution:



LESSON 29: EXERCISES

- 1) A random sample of 15 employees at a large company are surveyed about their incomes. Assume that incomes at the company are approximately normally distributed. The sample mean income is \$60,000, and the sample SD is \$25,000. You will find a 95% confidence interval (CI) for the population mean income at the company.

Use these hints about the t distribution on 14 degrees of freedom (df):



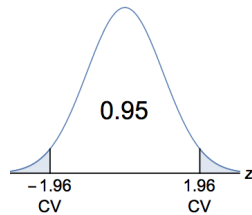
- a) Why do the methods of this Lesson apply to this problem?
- b) **Write the 95% CI** in the form $\mu = \bar{x} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula.
- c) **Write the 95% CI** in the form (lower limit, upper limit).
- d) **Interpret** the CI.

LESSON 30: EXERCISES

- 1) A magician's coin is flipped 200 times. It comes up heads 110 times. You will find a 95% confidence interval (CI) for the probability that the coin comes up heads on a flip.

Round off values of \hat{p} , \hat{q} , and E to three decimal places.

Use these hints about the z distribution:



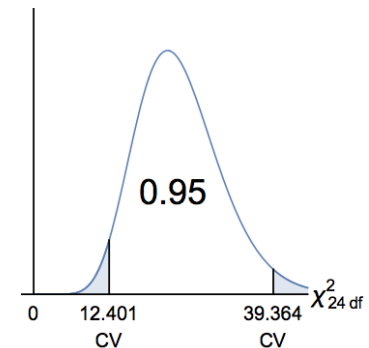
- a) Find the sample proportion of heads, \hat{p} .
- b) Find the sample proportion of tails, \hat{q} .
- c) **Verify** that normal approximations are appropriate in this problem.
- d) **Write the 95% CI** in the form $p = \hat{p} \pm E$. Clearly show how E is obtained by plugging into an appropriate formula.
- e) **Write the 95% CI** in the form (lower limit, upper limit).
- f) **Interpret** the CI.
- g) Based on this analysis, is there sufficient evidence against the claim that the coin is fair?
- h) We want to know the probability that the coin comes up heads on a flip. **Find the required sample size n** that would give us a margin of error of 0.05 for a 95% confidence interval (CI) for this probability. Clearly show how this is obtained by plugging into an appropriate formula (the one giving us a conservative estimate).
- i) Is your answer to h) more than, or less than, 200 [flips]? Compare the margin of error in h) with the one from d).

LESSON 31: EXERCISES

- 1) A wrench produced by a company is supposed to have size 1 inch. The wrench sizes produced by the company are approximately **normally distributed**. We randomly select 25 wrenches. The sample variance is 0.0025 square inches.

- a) We will find confidence intervals (CIs) for the population variance and SD of the wrench sizes produced by the company. Why do the methods of this Lesson apply to this problem?
- b) **Find** a 95% confidence interval (CI) for the population variance of the wrench sizes produced by the company. Clearly show how the limits are obtained by plugging into an appropriate formula. **Write** the CI in either the form lower limit $< \sigma^2 <$ upper limit or (lower limit, upper limit). **Interpret** the CI.
- c) **Also find** a 95% confidence interval (CI) for the population SD of the wrench sizes produced by the company. Clearly show how the limits are obtained by plugging into an appropriate formula. **Write** the CI in either the form lower limit $< \sigma <$ upper limit or (lower limit, upper limit). **Interpret** the CI.

Use these hints about the χ^2 distribution on 24 degrees of freedom (df):



LESSON 32: EXERCISES

- 1) In a hypothesis test, which do we sometimes accept: H_0 or H_1 ?
- 2) A magician says that “4” is the luckiest number. The magician gives you a die.
 H_0 states that the die is a standard die (with a uniform distribution for its faces).
 H_1 states that the die is **not** a standard die. You roll the die twice, and it comes up a “4” both times. The probability that a standard die rolled twice comes up a “4” both times by chance is about 0.028. Let’s take this as our P -value.
- a) If the significance level $\alpha = 0.05$, would we reject H_0 , based on this P -value?
 - b) If the significance level $\alpha = 0.01$, would we reject H_0 , based on this P -value?
- 3) Refer to Examples 3 and 5. A magician’s coin will be flipped 100 times. H_0 states that the coin is **fair**. H_1 states that the coin is **not fair**. Let’s say we observe 62 heads among the 100 flips. Let X = the number of heads in 100 flips of a fair coin. Use the significance level: $\alpha = 0.05$.
- a) Using a one-tailed P -value analysis,

$$\begin{aligned} \text{One-tailed } P\text{-value} &= P(X \geq 62) \\ &= P(62) + P(63) + P(64) + \dots + P(100) \\ &\approx 0.0105, \text{ or } 1.05\% \end{aligned}$$

Do we decide to **reject** or **not reject** H_0 ? What do we **conclude** about the assumption that the coin is fair?
 - b) Using a two-tailed P -value analysis,

$$\begin{aligned} \text{Two-tailed } P\text{-value} &= P(X \geq 62 \text{ or } X \leq 38) \\ &\approx 0.0210, \text{ or } 2.10\% \end{aligned}$$

Do we decide to **reject** or **not reject** H_0 ? What do we **conclude** about the assumption that the coin is fair?
 - c) A 95% confidence interval (CI) for p , the probability of heads, is: (0.525, 0.715). Do we **reject** or **not reject** H_0 in a two-tailed hypothesis test? What do we **conclude** about the assumption that the coin is fair?

- 4) Repeat Exercise 3), but use the significance level: $\alpha = 0.01$.

A magician’s coin will be flipped 100 times. H_0 states that the coin is **fair**. H_1 states that the coin is **not fair**. Let’s say we observe 62 heads among the 100 flips. Let X = the number of heads in 100 flips of a fair coin. Use the significance level: $\alpha = 0.01$.

- a) Using a one-tailed P -value analysis,

$$\begin{aligned} \text{One-tailed } P\text{-value} &= P(X \geq 62) \\ &= P(62) + P(63) + P(64) + \dots + P(100) \\ &\approx 0.0105, \text{ or } 1.05\% \end{aligned}$$

Do we decide to **reject** or **not reject** H_0 ? What do we **conclude** about the assumption that the coin is fair?

- b) Using a two-tailed P -value analysis,

$$\begin{aligned} \text{Two-tailed } P\text{-value} &= P(X \geq 62 \text{ or } X \leq 38) \\ &\approx 0.0210, \text{ or } 2.10\% \end{aligned}$$

Do we decide to **reject** or **not reject** H_0 ? What do we **conclude** about the assumption that the coin is fair?

- c) A 99% confidence interval (CI) for p , the probability of heads, is: (0.495, 0.745). Do we decide to **reject** or **not reject** H_0 ? What do we **conclude** about the assumption that the coin is fair?