LESSON 6: SYMMETRY, SKEWNESS, and MODALITY
What is the Shape of a Distribution?

PART A: THE SHAPE OF A DISTRIBUTION

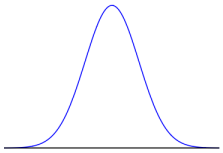
The distribution of a variable consists of its **possible values**, together with their **frequencies** or **relative frequencies** (later, we will use **probabilities**).

The shape of a distribution refers to the shape of a frequency or relative frequency **histogram** for **quantitative** data. A shape may be described by its symmetry, skewness, and/or modality.

PART B: SYMMETRIC DISTRIBUTIONS

Example 1 (Symmetric, Bell-Shaped Distribution)

The bell curve below is perfectly symmetric, because it can be divided into two halves (a left half and a right half) that are **mirror images** of each other. Think of it as a **"smoothed-out" histogram**.



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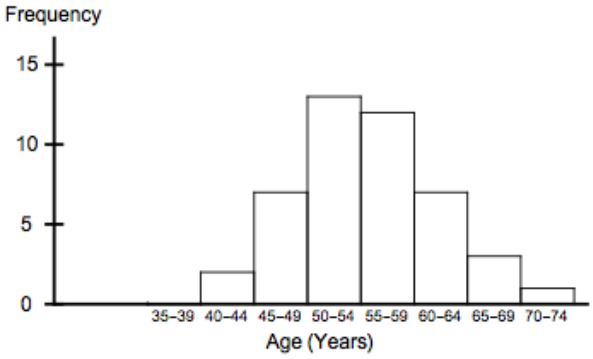
Example 2 (Approximately Symmetric Distribution: Presidents' Ages)

The **frequency histogram** below for presidents' ages may be called approximately symmetric, though that can be a subjective judgment.



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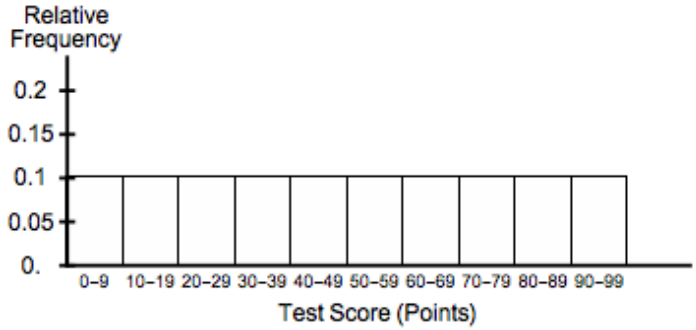
Example 3 (Symmetric, Uniform Distribution)

The **frequency histogram** of test scores below is uniform (or flat). Uniform distributions are **symmetric**.



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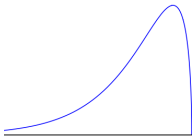
PART C: SKEWED DISTRIBUTIONS

A skewed distribution is an asymmetric (non-symmetric) distribution that has a long tail.

Example 4 (Left-Skewed Distribution)

The distribution below is skewed to the left (or is left-skewed) because it has a **long tail** extending to the **left**.



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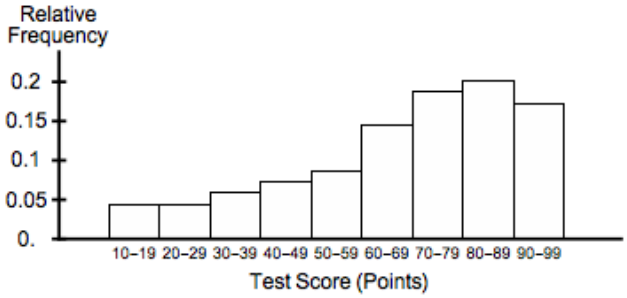
Example 5 (Left-Skewed Distribution: Test Scores)

The distribution of test scores below is **left-skewed**.



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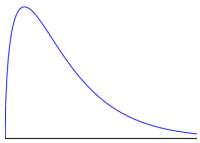
Think About It

In Example 5, if students are allowed to drop out of the course, what is likely to happen to the distribution of scores on future tests?

Example 6 (Right-Skewed Distribution)

The distribution below is skewed to the right (or is right-skewed) because it has a **long tail** extending to the **right**.



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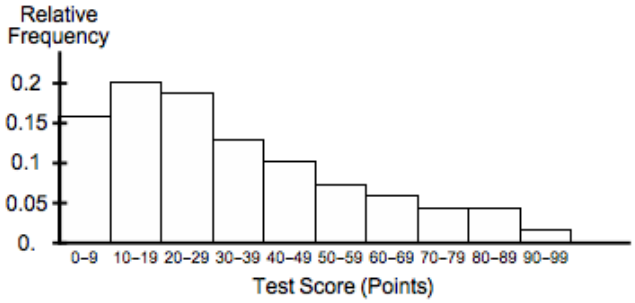
Example 7 (Right-Skewed Distribution: Scores on a Hard Test)

The distribution of test scores below is **right-skewed**.
It may have been a very difficult test!



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PART D: MODALITY

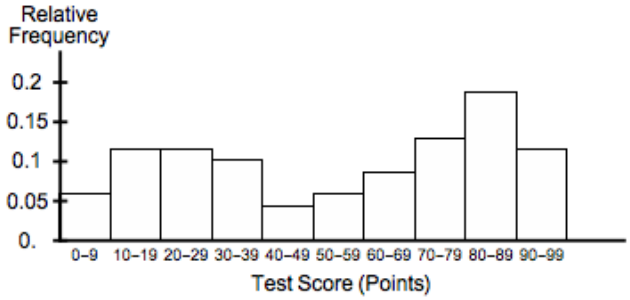
All but one of the distributions in Examples 1-7 were unimodal, meaning they had one mode (or "hump").

- The exception was Example 3. Flat, **uniform** distributions have **no mode**.

- Later, we will have a different definition of a "mode" for raw data (a list of values).

Example 8 (Bimodal Distribution)

The distribution of test scores below is bimodal, meaning it has two modes (or "humps").



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Think About It

What could explain this bimodal distribution in Example 8?

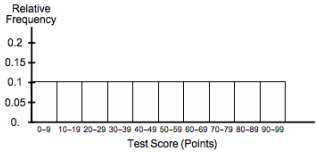
A multimodal distribution has more than two modes.

PART E: DESCRIBING DISTRIBUTION SHAPES (SUMMARY)

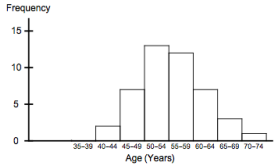
Example 9 (Describing Distribution Shapes)

Describe these distribution shapes.

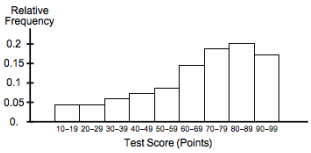
- a)



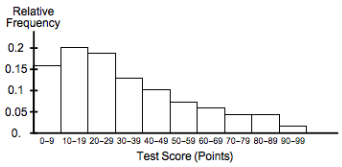
- b)



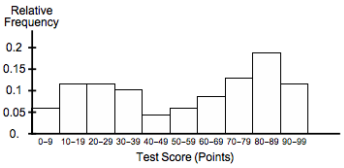
- c)



- d)



- e)



§ Solution

- a) This distribution is uniform and symmetric, and it has no mode.
- b) This distribution is unimodal and is approximately bell-shaped and symmetric.
- c) This distribution is unimodal and left-skewed.
- d) This distribution is unimodal and right-skewed.
- e) This distribution is bimodal and is neither symmetric nor skewed.

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