SOME BASIC TRIGONOMETRY - KNOW THIS!

FUNDAMENTAL TRIGONOMETRIC IDENTITIES (IDs)

Memorize these in both "directions" (left-to-right and right-to-left).

Reciprocal Identities

$$\csc(x) = \frac{1}{\sin(x)} \qquad \sin(x) = \frac{1}{\csc(x)}$$

$$\sec(x) = \frac{1}{\cos(x)} \qquad \cos(x) = \frac{1}{\sec(x)}$$

$$\cot(x) = \frac{1}{\tan(x)} \qquad \tan(x) = \frac{1}{\cot(x)}$$

Warning: The reciprocal of $\sin(x)$ is $\csc(x)$, not $\sec(x)$.

Note: We typically treat "0" and "undefined" as reciprocals when we are dealing with basic trigonometric functions. Your algebra teacher will not want to hear this, though!

Quotient Identities

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \quad \text{and} \quad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

Pythagorean Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$\tan^2(x) + 1 = \sec^2(x)$$

Tip: The 2nd and 3rd IDs can be obtained by dividing both sides of the 1st ID by $\sin^2(x)$ and $\cos^2(x)$, respectively.

Tip: The squares of $\csc(x)$ and $\sec(x)$, which have the "Up-U, Down-U" graphs, are all alone on the right sides of the last two IDs. They can never be 0 in value. (Why is that? Look at the left sides.)

Cofunction Identities

If x is measured in radians, then:

$$\sin(x) = \cos\left(\frac{\pi}{2} - x\right)$$
$$\cos(x) = \sin\left(\frac{\pi}{2} - x\right)$$

We have similar relationships for tangent and cotangent - and for secant and cosecant; remember that they are sometimes undefined.

Think: Cofunctions of complementary angles are equal.

Even / Odd (or Negative Angle) Identities

Among the six basic trigonometric functions, cosine (and its reciprocal function, secant) are even:

$$\cos(-x) = \cos(x)$$
$$\sec(-x) = \sec(x), \text{ when both sides are defined}$$

The other four (sine and cosecant, tangent and cotangent) are odd:

$$\sin(-x) = -\sin(x)$$
$$\csc(-x) = -\csc(x), \text{ when both sides are defined}$$

$$\tan(-x) = -\tan(x), \text{ when both sides are defined}$$
$$\cot(-x) = -\cot(x), \text{ when both sides are defined}$$

Note: If f is an **even** function (such as cosine), then the graph of $y = f(x)$ is symmetric about the ***y*-axis**.

Note: If f is an **odd** function (such as sine), then the graph of $y = f(x)$ is symmetric about the **origin**.