QUIZ 2 SOLUTIONS

(LESSONS 11-18: DISCRETE PROBABILITY)
STAT C1000 – FALL 2026 – KUNIYUKI
100 POINTS TOTAL

No notes or books allowed. A scientific calculator is allowed. Simplify as appropriate.
You do <u>not</u> have to reduce fractions. For example, 10/20 does <u>not</u> have to be rewritten as ½.
Do <u>not</u> leave decimal parts in fractions or fractions in fractions, such as .1/.2 or (1/3)/(2/3).

1) (12 points). Two standard six-sided dice are rolled. One die is red; the other is green. The "total" of the two dice is the sum of the numbers on the dice. Write your answers as **exact fractions**.

• a) What is the probability of getting a 4 on the red die and a 4 on the green die? Your answer will be one fraction. (2 points)

$$\begin{aligned} & P(\text{4 on red } \textbf{and } \text{4 on green}) \\ = \; & P(\text{4 on red}) \cdot P(\text{4 on green}) \quad \text{(by independence)} \\ = \; & \frac{1}{6} \cdot \frac{1}{6} \\ = \; & \boxed{\frac{1}{36}} \end{aligned}$$

Also, observe that **only one** of the 36 equally likely ordered (red, green) pairs suggested by the grid below corresponds to "4 on red **and** 4 on green."

	green	1	2	3	4	5	6
red							
1							
2							
3							
4					x		
5							
6							

• b) What is the probability that the total is 6? Show work by using a grid as in class (you don't have to draw lines) or listing ordered pairs. (5 points)

$$P(\text{total is 6}) \; = \; \boxed{\frac{5}{36}}$$

Observe that **five** of the 36 equally likely ordered (red, green) pairs suggested by the grid below correspond to "total is 6."

	green	1	2	3	4	5	6
red							
1						x	
2					x		
3				x			
4			x				
5		x					
6							

Ordered pairs: (5,1), (4,2), (3,3), (2,4), (1,5).

• c) What is the probability of getting a 2 or a 5 on the red die or a 3 or a 4 on the green die? Your answer will be one fraction. Show work by using a grid as in class (you don't have to draw lines) or listing ordered pairs. (5 points)

$$P(\text{2 or 5 on red } \textbf{or} \text{ 3 or 4 on green}) = \boxed{\frac{20}{36}, \text{ or } \frac{5}{9}}$$

Observe that **20** of the 36 equally likely ordered (red, green) pairs suggested by the grid below correspond to "2 or 5 on red **or** 3 or 4 on green." **Remember to not double-count the "x"s.**

	green	1	2	3	4	5	6
red							
1				x	x		
2		x	x	x	x	x	x
3				x	x		
4				x	x		
5		x	x	x	x	x	x
6				x	x		

Ordered pairs: (2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(1,3), (3,3), (4,3), (6,3),
(1,4), (3,4), (4,4), (6,4).

You could work out the 20 by taking: 6 + 6 + 6 + 6 – 4.
The idea is to add the six entries of "x"s from each of the two full rows and each of the two full columns of "x"s but then subtracting the 4 ordered pairs in the overlap that's being double-counted.

2) (5 points). Two popular soda drinks are Poke and Cepsi. According to a study, 60% of American adults like Poke, 45% like Cepsi, and 30% like both. (Assume that the study is correct.) What is the probability that a randomly selected American adult likes Poke or Cepsi? Write your answer as a decimal, a percent, or a fraction.

Use the **General Addition Rule**.

$$P(\text{likes Poke } \textbf{or} \text{ likes Cepsi}) = P(\text{likes Poke}) + P(\text{likes Cepsi}) - P(\text{likes Poke } \textbf{and} \text{ likes Cepsi})$$
$$= 0.60 + 0.45 - 0.30$$
$$= \boxed{0.75, \text{ or } 75\%, \text{ or } \frac{75}{100}, \text{ or } \frac{3}{4}}$$

3) (20 points). Professor Potter teaches two classes – a morning class and an evening class. All of Potter's 85 students take a test, and a student can either pass or fail the test. Consider the following two-way frequency (or contingency) table. Write your answers as **exact fractions**.

		Test Result		
		Pass	Fail	Total
Class	Morning	30	20	50
	Evening	25	10	35
	Total	55	30	85

• a) What is the **conditional probability** that a randomly selected student of Potter's is a morning student, **given that** the person passed the test? (5 points)

		Test Result		
		Pass	Fail	Total
Class	Morning	30	20	50
	Evening	25	10	35
	Total	55	30	85

$$P(\text{ morning} \mid \text{passed}) = \frac{\#(\text{passed } \textbf{and} \text{ morning})}{\#(\text{passed})}$$

$$= \boxed{\frac{30}{55}, \text{ or } \frac{6}{11}}$$

• b) What is the **conditional probability** that a randomly selected student of Potter's passed the test, **given that** the person is a morning student? (5 points)

		Test Result		
		Pass	Fail	Total
Class	Morning	30	20	50
	Evening	25	10	35
	Total	55	30	85

$$P(\text{ passed} \mid \text{morning}) = \frac{\#(\text{morning } \textbf{and} \text{ passed})}{\#(\text{morning})}$$

$$= \boxed{\frac{30}{50}, \text{ or } \frac{3}{5}}$$

• c) What is the probability that a randomly selected student of Potter's is in the evening class **or** passed the test? Your answer will be one fraction. (5 points)

$$P(\text{evening } \mathbf{or} \text{ passed}) = \frac{\#(\text{evening } \mathbf{or} \text{ passed})}{N}$$

$$= \boxed{\frac{65}{85}, \text{ or } \frac{13}{17}}$$

For the numerator, 65, we can either **add** the **boldfaced red** entries below, or we could simply **subtract** 20 from 85.

		Test Result		
		Pass	Fail	Total
Class	Morning	**30**	20	50
	Evening	**25**	**10**	35
	Total	55	30	**85**

• d) What is the probability that a randomly selected student of Potter's is in the evening class **and** passed the test? Your answer will be one fraction. (5 points)

$$P(\text{evening } \mathbf{and} \text{ passed}) = \frac{\#(\text{evening } \mathbf{and} \text{ passed})}{N}$$

$$= \boxed{\frac{25}{85}, \text{ or } \frac{5}{17}}$$

		Test Result		
		Pass	Fail	Total
Class	Morning	30	20	50
	Evening	**25**	10	35
	Total	55	30	**85**

4) (4 points). Four standard six-sided dice are rolled. What is the probability that all four dice come up "6"s? Your answer will be one fraction. **Show work** by showing how you got your answer!

$$P(\text{four "6"s}) = [P(6)]^4 \text{ (by independence)} = \left[\frac{1}{6}\right]^4 = \boxed{\frac{1}{1296}}$$

5) (8 points). Three cards are randomly drawn from a standard 52-card deck **without replacement** (each drawn card is immediately, permanently removed from the deck). What is the probability that all three cards are Queens? Write your answer as an exact fraction and also round it as a decimal to three significant figures. **Show work** by writing fractions and what you do with them! Hint: There are originally four Queens in the deck of cards.

Because we are drawing cards **without replacement**, previous draws affect probabilities on later draws, and the draws are **dependent** trials.

$$P(\text{Queen}-1\text{st } \textbf{and } \text{Queen}-2\text{nd } \textbf{and } \text{Queen}-3\text{rd})$$

$$= P(\text{Queen}-1\text{st}) \cdot P(\text{Queen}-2\text{nd} \mid \text{Queen}-1\text{st}) \cdot P(\text{Queen}-3\text{rd} \mid \text{Queen}-1\text{st and}-\text{Queen 2nd})$$

$$= \underbrace{\frac{4}{52}}_{\substack{52 \text{ cards} \\ 4 \text{ Queens}}} \cdot \underbrace{\frac{3}{51}}_{\substack{51 \text{ cards} \\ 3 \text{ Queens}}} \cdot \underbrace{\frac{2}{50}}_{\substack{50 \text{ cards,} \\ 2 \text{ Queens}}}$$

$$= \boxed{\frac{24}{132{,}600}, \text{ or } \frac{1}{5525}}$$

$$\approx \boxed{0.000181}$$

6) (4 points). A student takes three classes. Let X = the number of classes the student will pass. The student gives the following **incomplete** probability distribution for X. Find the value of a to complete the probability distribution.

Value (x)	**Probability** $P(x)$
0	0.100
1	a
2	0.350
3	0.400

$a = 1 - 0.100 - 0.350 - 0.400 = \boxed{0.150}$.

7) (12 points). You and your family first pay $500 for a life insurance policy for the year. If you die during the year, your family gets $20,000. If you do not die during the year, your family gets nothing (beyond your presence, at least). Let X = you and your family's net monetary gain as a result of the decision to purchase the policy. Based on research, you assume that your probability of dying during the year is 0.020. Assuming this is correct, find $E(X)$ and **interpret** it, as we did in class. (Hint: Consider many customers in a situation similar to yours.) Also, fill out the table:

Outcome for the year	**Value** (x)	**Probability** $P(x)$
You live.	$\boxed{-\$500}$	$1-0.020=\boxed{0.980}$
You die.	$\$20{,}000-\$500=\boxed{\$19{,}500}$	$\boxed{0.020}$

$$E(X) \text{ or } \mu = \sum P(x)\cdot x$$
$$= (0.980)(-\$500)+(0.020)(\$19{,}500)$$
$$= \boxed{-\$100}$$

Over many customers in a similar situation as you, the long-run average net **loss** for each family is about $100 per family.

8) (15 points). Showing some work or notation may help with partial credit. None of the answers is "one." Each answer is a single number (of ways).

• a) You have nine candy bars in your Halloween bowl; all of them are different. A scary ghost comes to your door. You decide to give the ghost four of your candy bars. How many ways can you choose the four candy bars for the ghost? The order of the chosen candy bars does not matter. (5 points)

$${}_9C_4, \text{ or } \binom{9}{4} = \frac{9!}{4!(9-4)!} = \boxed{126} \text{ ways}$$

• b) Your neighbor also has nine candy bars in a Halloween bowl; all of them are different. Nine children form a line outside your neighbor's door. Your neighbor will give one candy bar to each child. How many ways can the nine candy bars be given to the nine children? (5 points)

$9! = \boxed{362{,}880}$ ways

• c) Billy Bonka makes nine different flavors of candy bars. You buy Billy Bonka bars at a large store. You decide to eat one Billy Bonka candy bar each day for the next five days. (Assume that every flavor is available to you on every day.) How many ways can you eat five Billy Bonka candy bars for the next five days if you only care about flavors and days? (5 points)

$9^5 = \boxed{59{,}049}$ ways

9) (5 points). A student answers all six questions on a multiple-choice math quiz. Each question has three possible options: "A," "B," or "C," only one of which is correct. The student guesses randomly on all questions. The random variable is the number of questions the student gets correct. As in class, give the distribution (including the type of distribution and the values of the two parameters) that best describes the random variable.

$$X \sim \boxed{\text{Bin}\left(n = 6,\ p = \frac{1}{3}\right)}$$

10) (15 points; 5 points each). It turns out that the random variable described in 9) above has the following probability distribution table.

Value (x)	**Probability** $P(x)$
0	0.088
1	0.263
2	0.329
3	0.219
4	0.082
5	0.016
6	0.001

Find the indicated probabilities regarding the number of questions the student gets correct. Write your answers to three decimal places. Showing work can help with partial credit; for example, rewriting using an inequality or rewriting as a sum or difference of probabilities.

- a) P(at most 2)

$$\begin{aligned} P(X \le 2) &= P(0) + P(1) + P(2) \\ &\approx 0.088 + 0.263 + 0.329 \\ &\approx \boxed{0.680} \end{aligned}$$

- b) P(at least 5)

$$\begin{aligned} P(X \ge 5) &= P(5) + P(6) \\ &\approx 0.016 + 0.001 \\ &\approx \boxed{0.017} \end{aligned}$$

- c) P(more than 3)

$$\begin{aligned} P(X > 3) &= P(4) + P(5) + P(6) \\ &\approx 0.082 + 0.016 + 0.001 \\ &\approx \boxed{0.099} \end{aligned}$$