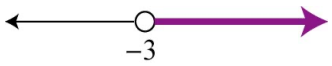
QUIZ 1B - SOLUTIONS

(CHAPTER 1: FUNCTIONS)
MATH 141 – FALL 2026 – KUNIYUKI
68 POINTS TOTAL, BUT 60 POINTS = 100%

1) (4 points). Write the domain of f, where $f(x)=\dfrac{1}{\sqrt{x+3}}+\sqrt[3]{x-7}$, using interval form (the form using parentheses and/or brackets).

- $f(x)$ is real $\Leftrightarrow$ $x+3>0$ (We forbid 0 denominators.) $\Leftrightarrow$ $x>-3$.
- $\sqrt[3]{x-7}$, which is an odd root of a polynomial, imposes no restrictions.

Here is a graph of the domain, $\text{Dom}(f)$:



In interval form, $\text{Dom}(f)=\boxed{(-3,\infty)}$.

2) (3 points). Find **and box in** the x-intercept(s) (if any) of the graph of $y=\dfrac{\sqrt{x^2-25}}{x-5}$ in the usual xy-plane. If there are none, write "NONE."

To find x-intercepts, we substitute ("plug in") $y=0$. Find the real solutions of: $0=\dfrac{\sqrt{x^2-25}}{x-5}$. That is, we want the real zeros of f, where $f(x)=\dfrac{\sqrt{x^2-25}}{x-5}$.

$$f(x)=0 \quad\Leftrightarrow\quad \left(\sqrt{x^2-25}=0\right) \text{ and } \left(x-5\neq 0\right)$$
$$\Leftrightarrow\quad \left(x^2-25=0\right) \text{ and } \left(x\neq 5\right)$$

The real zeros of x^2-25 are 5 and -5. We can use the Quadratic Formula (QF) or:

Square root method

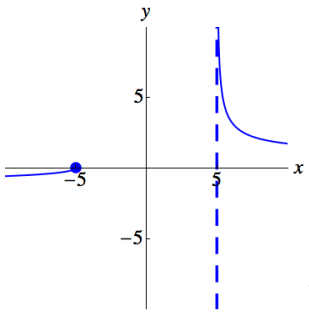
$$x^2-25=0$$
$$x^2=25$$
$$x=\pm 5 \quad (x=5 \text{ or } x=-5)$$

Factoring method

$$x^2-25=0$$
$$(x+5)(x-5)=0 \quad [(x+5)=0 \text{ or } (x-5)=0]$$
$$x=\pm 5 \quad [x=-5 \text{ or } x=5]$$

However, "5" must be **rejected**, because it is **not in the domain** of f. We only take -5.

The only x-intercept is at $\boxed{(-5,0)}$.



Note: Here is the graph of $y=f(x)$.

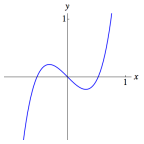
3) (1 point). The graph of $y = 2x^5 + 3x^3 - x$ is symmetric about the (Box in one:)

x-axis y-axis $\boxed{\text{origin}}$ (none of these)

Let $f(x) = 2x^5 + 3x^3 - x$. $\text{Dom}(f) = \mathbb{R}$. f is an **odd** function, because $\forall x \in \mathbb{R}$,

$$f(-x) = 2(-x)^5 + 3(-x)^3 - (-x) = -2x^5 - 3x^3 + x = -(2x^5 + 3x^3 - x) = -f(x)$$

A polynomial (with no nonzero constant term) written in descending powers of x where all exponents on x are **odd** represents an **odd** function of x. The graph of $y = f(x)$:



Since f is an **odd** function, the graph of $y = f(x)$ is symmetric about the **origin**.

4) (1 point). If the point $(-5, 4)$ lies on the graph of $y = f(x)$, what other point must also lie on the graph if f is an **even** function?

$\boxed{(5, 4)}$, because the graph must be **symmetric about the *y*-axis**.

5) (6 points total). If the point $(1, -5)$ lies on the graph of $y = f(x)$, where f is a one-to-one function, what point must then lie on the graph of …

a) … $y = f(x-1) + 4$?

$\boxed{(2, -1)}$, after the point $(1, -5)$ is shifted 1 unit **right** and 4 units **up**.

b) … $y = -f(x)$?

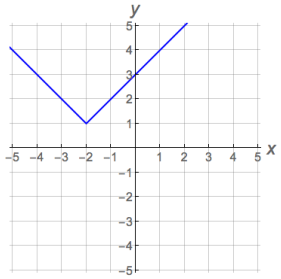
$\boxed{(1, 5)}$, because the point $(1, -5)$ is **reflected about the *x*-axis**, thus yielding the point $(1, 5)$.

c) … $y = f^{-1}(x)$?

$\boxed{(-5, 1)}$, the **reflection** of the point $(1, -5)$ about the **line** $y = x$.

We **switched** the x- and y-coordinates.

6) (4 points). The graph below is obtained by taking a basic graph from Section 1.3 and applying rigid transformations. Find an equation for the graph.



Fill in the blank: $y =$ $\boxed{|x+2|+1}$.

This is the "V" graph of $y = |x|$ shifted **to the left** by 2 units and **raised up** by 1 unit.

7) (1 point). Evaluate $[\![-7.1]\!]$. (This is the same as $\lfloor -7.1 \rfloor$.)

$\boxed{-8}$; the greatest integer (or floor) function rounds down.

-8 is the greatest integer that **does not exceed** -7.1.

8) (6 points). Match the equations with their corresponding graphs by writing the appropriate letters in the blanks. The *x*- and *y*-axes are not necessarily scaled the same way.

The graph of $y = \dfrac{1}{x}$ is Graph **A**. (Hyperbola)

The graph of $y = \dfrac{1}{x^2}$ is Graph **E**. ("Volcano")

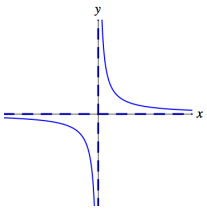
The graph of $y = |x|$ is Graph **D**. ("V" graph)

The graph of $y = \sqrt[3]{x}$ is Graph **C**. ("Lazy snake" / "S" graph)

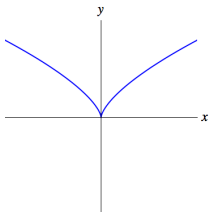
The graph of $y = x^{2/3}$ is Graph **B**. ("Bird beak/Tornado/Funnel")

The graph of $y = \sqrt{4 - x^2}$ is Graph **F**. (Upper semicircle)

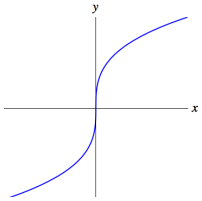
Graph A



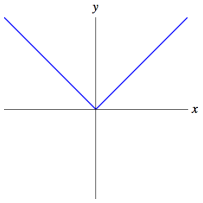
Graph B



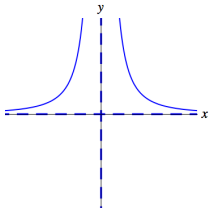
Graph C



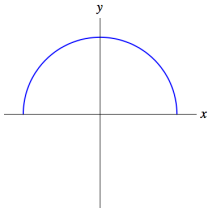
Graph D



Graph E



Graph F



• A and C are graphs of **odd** functions, while the others are graphs of **even** functions.

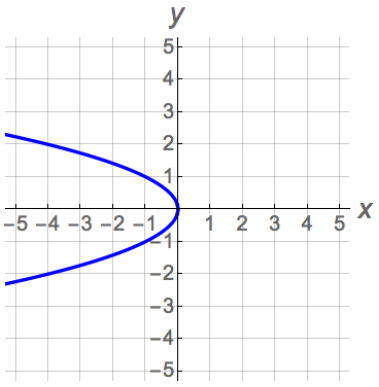
• The **domains** of the functions corresponding to A, B, C, D, E, and F are, respectively, $(-\infty, 0) \cup (0, \infty)$, $\mathbb{R}$, $\mathbb{R}$, $\mathbb{R}$, $(-\infty, 0) \cup (0, \infty)$, and $[-2, 2]$.

9) (2 points). Find functions *g* and *f* such that $(f \circ g)(x) = \sqrt[3]{x^2 + 1}$.

You may not use the identity function. Fill in the blanks:

$\boxed{g(x) = x^2 + 1,\ f(u) = \sqrt[3]{u}}$. There are many other possibilities.

10) (2 points). Graph $x = -y^2$ on the grid below.



The graph of $x = y^2$, a **rightward-opening parabola** with vertex at the origin, is **reflected about the *y*-axis**, so we obtain a **leftward-opening parabola** for $x = -y^2$.

11) (4 points). Let $f(x) = x^3 + 1$. What is $f^{-1}(x)$?

Answer: $f^{-1}(x) = \sqrt[3]{x-1}$.

Conceptual Approach. f **cubes** its input and **adds 1** to the result. f^{-1} **subtracts 1** from its input and takes **the cube root** of the result to undo f.

Mechanical Approach.

Step 1: Replace $f(x)$ with y.

$$y = x^3 + 1$$

Step 2: Switch x and y.

$$x = y^3 + 1$$

Step 3: Solve for y.

$$x - 1 = y^3$$
$$\sqrt[3]{x-1} = y$$
$$y = \sqrt[3]{x-1}$$

Step 4: Replace y with $f^{-1}(x)$.

$$f^{-1}(x) = \sqrt[3]{x-1}$$

12) Let $s(t) = t^3 - 2t$. Find the average rate of change of s from $t = 1$ to $t = 4$. Assume that t is time measured in seconds and $s(t)$ is the position of a particle measured in meters. (The particle is moving along a coordinate line.) Write the appropriate unit in your final answer. Show all work, as in class!

Note: You are finding the average velocity of the particle between $t = 1$ second and $t = 4$ seconds. (7 points)

s is a polynomial function, so the usual formula applies.

$$\frac{s(b)-s(a)}{b-a} = \frac{s(4)-s(1)}{4-1} = \frac{\left[(4)^3-2(4)\right]-\left[(1)^3-2(1)\right]}{3} = \frac{56-[-1]}{3} = \frac{57}{3} = \boxed{19 \frac{\text{meters}}{\text{second}}}$$

13) f is the function defined piecewise by: $f(x)=\begin{cases} x^2-1, & -2\le x<2 \\ 1-x, & x\ge 2 \end{cases}$. (18 points)

a) Evaluate $f(-3)$. (2 points)

$-3\notin \text{Dom}(f)$, so $f(-3)$ is $\boxed{\text{undefined}}$.

b) Evaluate $f(-2)$. (2 points)

$-2\le -2<2$, so use the **top** rule:

$$f(x)=x^2-1 \quad\Rightarrow\quad f(-2)=(-2)^2-1=4-1=\boxed{3}.$$

c) Evaluate $f(2)$. (2 points)

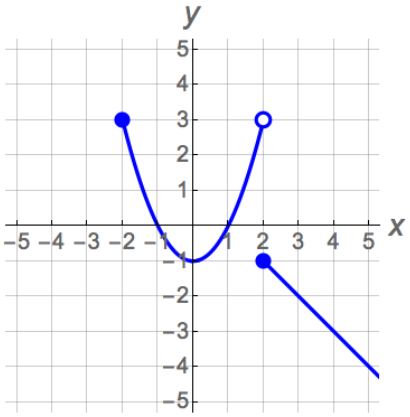
$2\ge 2$, so use the **bottom** rule:

$$f(x)=1-x \quad\Rightarrow\quad f(2)=1-(2)=\boxed{-1}.$$

d) Graph $y=f(x)$ on the grid below. Be accurate. Clearly indicate whether endpoints are included or excluded, as in class. (7 points)

• Locate **endpoints** on the graph (corresponding to endpoints of the subdomains) and determine if they are **included** in (or **excluded** from) the graph.

Rule	Left endpoints	Right endpoints	Shapes
x^2-1	$x=-2$ $y=(-2)^2-1=3$ See Part a). **Left endpoint:** $(-2,3)$ **Included** because of "$\le$" (weak inequality).	$x=2$ $y=(2)^2-1=3$ **Right endpoint:** $(2,3)$ **Excluded** because of "$<$" (strict inequality).	The parabola with equation $y=x^2-1$ is the graph of $y=x^2$ **shifted** 1 unit **down**. We take a piece of this graph.
$1-x$	$x=2$ $y=1-(2)=-1$ See Part b). **Left endpoint:** $(2,-1)$ **Included** because of "$\ge$" (weak inequality).	**(None)**	The graph of $y=1-x$, or $y=-x+1$, is the line with slope -1 and y-intercept at $(0,1)$. We take a piece of this graph.



e) Give the **domain** of the function f using interval form (the form with parentheses and/or brackets). (2 points)

$\text{Dom}(f) = \boxed{[-2, \infty)}$, the **union** of the given subdomains: $[-2, 2) \cup [2, \infty)$.

The domain gives the ***x*-coordinates** "picked up" by the graph of $y = f(x)$.

f) Give the **range** of the function f using interval form (the form with parentheses and/or brackets). (3 points)

$\text{Range}(f) = \boxed{(-\infty, 3]}$, which gives the ***y*-coordinates** "picked up" by the graph.

14) Let $f(x) = \sqrt{x}$. Simplify the difference quotient completely:

$$\frac{f(x+h) - f(x)}{h}$$

Show all work!
Hint: You will need to rationalize a numerator. (9 points)

• When you see $f(x+h)$, think "**substitution**."

$\forall h \neq 0,$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \quad \text{(Rationalize the numerator.)}$$

$$= \frac{\left(\sqrt{x+h} - \sqrt{x}\right)}{h} \cdot \frac{\left(\sqrt{x+h} + \sqrt{x}\right)}{\left(\sqrt{x+h} + \sqrt{x}\right)} = \frac{\left(\sqrt{x+h}\right)^2 - \left(\sqrt{x}\right)^2}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \frac{(x+h) - (x)}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \frac{\cancel{x} + h \cancel{- x}}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \frac{\overset{(1)}{\cancel{h}}}{\underset{(1)}{\cancel{h}}\left(\sqrt{x+h} + \sqrt{x}\right)} = \boxed{\frac{1}{\sqrt{x+h} + \sqrt{x}} \quad (h \neq 0)}$$

• As h approaches 0, this approaches $\frac{1}{2\sqrt{x}}$, which is $f'(x)$, the derivative of $f(x)$.