MIDTERM 2 SOLUTIONS

(CHAPTERS 2 AND 3: POLYNOMIAL, RATIONAL, EXP'L, LOG FUNCTIONS)
MATH 141 – FALL 2026 – KUNIYUKI
150 POINTS TOTAL: 44 FOR PART 1, AND 106 FOR PART 2

PART 1: USING SCIENTIFIC CALCULATORS (44 PTS.)

1) A profit function for a (cheap) statue-making company is given by: $p(x) = -200x^2 + 1200x - 200$ (in dollars), where x is the number of statues produced, and $x \geq 0$. Write units! (10 points total)

a) Write and use a formula we used in class to find the production level (that is, the number of statues produced) that will lead to the maximum profit. (4 points)

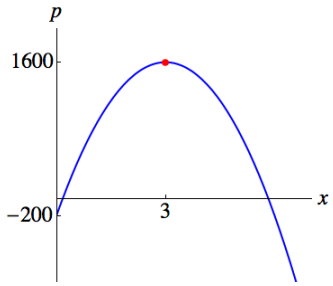
The graph of $p(x)$ versus x is a piece of a parabola opening downward. We want the x-coordinate of the vertex (the maximum point) of the parabola.

$$x = -\frac{b}{2a} = -\frac{(1200)}{2(-200)} = \boxed{3 \text{ statues}}$$ will lead to the maximum profit.

b) What is the maximum profit? (4 points)

The corresponding maximum profit is given by:

$$p(3) = -200(3)^2 + 1200(3) - 200 = \boxed{\$1600}.$$



c) What is the profit when no statues are produced? (2 points)

$p(0) = \boxed{-\$200}$. This implies a net loss of \$200; these may be fixed costs.

2) Simplify $\dfrac{1}{3-7i}$ by writing the quotient in standard form. (5 points)

Multiply the numerator and denominator by the **complex conjugate** of the denominator.

$$\frac{1}{3-7i} = \frac{1}{(3-7i)} \cdot \frac{(3+7i)}{(3+7i)} = \frac{3+7i}{9-49i^2} = \frac{3+7i}{9+49} = \frac{3+7i}{58} = \frac{3}{58} + \frac{7i}{58} = \boxed{\frac{3}{58} + \frac{7}{58}i}$$

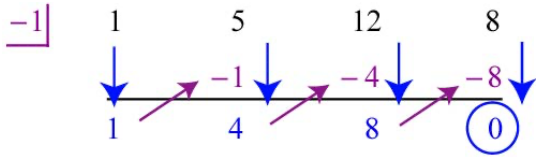
or use: $a^2 + b^2 = (3)^2 + (7)^2 = 9 + 49 = 58$

3) Consider $f(r) = r^3 + 5r^2 + 12r + 8$ in parts a) and b) below.
Hint: One of the zeros is -1. (16 points total)

a) Write the two other complex zeros of f in simplest, standard form. Show all work, as in class. Box in your answers! (13 points)

By the **Factor Theorem**, because -1 is a zero, $(r+1)$ is a factor of $f(r)$.

Use **Synthetic Division** to check this and to help us factor $f(r)$.



Therefore, $f(r) = (r+1)(r^2 + 4r + 8)$.

Find the zeros (roots) of the quadratic factor, $r^2 + 4r + 8$.

- We have: $a = 1,\ b = 4,\ c = 8$
- The **discriminant** is: $b^2 - 4ac = (4)^2 - 4(1)(8) = 16 - 32 = \mathbf{-16}$
- $\mathbf{-16}$ is **not** a perfect square, and GCF = 1. We **cannot** factor $r^2 + 4r + 8$ over $\mathbb{Z}$ using the "Guess Method."
- We can find the zeros (roots) by using the **Quadratic Formula (QF)**:

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(4) \pm \sqrt{\mathbf{-16}}}{2(1)} = \frac{-4 \pm 4i}{2} = \frac{-4}{2} \pm \frac{4i}{2} = \boxed{-2 \pm 2i}$$

- We already verified that -1 was a zero.

b) Write the polynomial $f(r)$ as a product of three linear factors over $\mathbb{C}$, the set of complex numbers. We basically want the Linear Factorization Theorem (LFT) Form of the factorization. (3 points)

$$\boxed{f(r) = (r+1)\left[r - (-2+2i)\right]\left[r - (-2-2i)\right], \text{ or } (r+1)\left[r+2-2i\right]\left[r+2+2i\right]}$$

4) An exponential growth model for the population of Fredonia is given by: $P(t) = P_0 e^{0.0471t}$, where $P(t)$ is the population t years after January 1, 2020. The population of Fredonia was 64,000 people on January 1, 2020. (9 points total)

a) In how many years after January 1, 2020 will the population of Fredonia be 750,000 people? Give **both** an **exact** answer (which may look ugly; you don't have to simplify it) and an **approximate** answer rounded off to three significant digits. Write units! (8 points)

Model is $P(t) = P_0 e^{0.0471t}$

Solve $750{,}000 = 64{,}000 e^{0.0471t}$ for t

$$\frac{750{,}000}{64{,}000} = e^{0.0471t}$$

$$\frac{375}{32} = e^{0.0471t}$$

$$\ln\left(\frac{375}{32}\right) = \ln\left(e^{0.0471t}\right)$$

$$\ln\left(\frac{375}{32}\right) = 0.0471t$$

$$t = \frac{\ln\left(\frac{375}{32}\right)}{0.0471}$$

$$\text{or } \frac{\ln\left(\frac{375}{32}\right)}{\frac{471}{10{,}000}}$$

$$\text{or } \frac{10{,}000\left[\ln(375) - \ln(32)\right]}{471} \text{ years (exactly)}$$

$t \approx 52.3$ years (approximately)

b) In what year will the population of Fredonia be 750,000 people? You may use part a). (1 point)

The population of Fredonia will be 750,000 people in the year $\boxed{2072}$.

5) Approximate $\log_4(9753)$ to four decimal places. Show work by using a change-of-base formula we have discussed in class. (4 points)

Use the **Change of Base Formula**: $\log_4(9753) = \frac{\ln(9753)}{\ln(4)}$ $\left[\text{or } \frac{\log(9753)}{\log(4)}\right] \approx \boxed{6.6258}$.

PART 2: NO CALCULATORS ALLOWED! (106 POINTS)

6) Use Long Division to perform the division: $\dfrac{9x^4 - x^2 + 7x + 1}{3x^2 + 2x}$.

Write your answer in the form: $\left(\text{polynomial}\right) + \left(\text{proper rational expression}\right)$.

(15 points)

The terms in red are deleted after their opposites are written in the subtraction process.

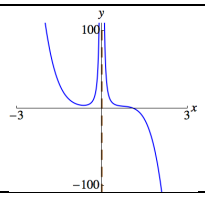
$$
\begin{array}{r}
 & 3x^2 - 2x + 1 \\
\hline
3x^2 + 2x \;\big)\; 9x^4 + 0x^3 - x^2 + 7x + 1 \\
\quad \cancel{9x^4 + 6x^3} \\
\quad -9x^4 - 6x^3 \\
\hline
\qquad -6x^3 - x^2 + 7x + 1 \\
\qquad \cancel{-6x^3 - 4x^2} \\
\qquad 6x^3 + 4x^2 \\
\hline
\qquad\qquad 3x^2 + 7x + 1 \\
\qquad\qquad \cancel{3x^2 + 2x} \\
\qquad\qquad -3x^2 - 2x \\
\hline
\qquad\qquad 5x + 1
\end{array}
$$

$5x + 1$ has degree 1, which is less than 2, the degree of $3x^2 + 2x$ (the divisor). Therefore, $5x + 1$ is an appropriate remainder.

We obtain: $\boxed{3x^2 - 2x + 1 + \dfrac{5x + 1}{3x^2 + 2x}}$

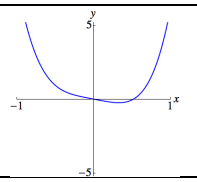
7) Fill in each blank below with ∞ or $-\infty$. (4 points total; 2 points each)

a) If $f(x) = -3x^5 + 4x^2 - 1 + \frac{1}{x^2}$, then $\lim_{x \to -\infty} f(x) =$ $\boxed{\infty}$



$\lim_{x \to -\infty} \frac{1}{x^2} = 0$, so focus on the polynomial part. Its leading term is $-3x^5$, so the graph of $y = f(x)$ is a "falling snake in the long run." The graph "shoots up" without bound to the left.

b) If $g(x) = 7x^4 - x$, then $\lim_{x \to \infty} g(x) =$ $\boxed{\infty}$



The leading term is $7x^4$, so the graph of $y = g(x)$ is an "upward-opening bowl in the long run." The graph "shoots up" without bound to the right.

8) Write the list of the possible rational zeros of f, where $f(x) = 2x^4 - 11x^2 + x - 3$, based on the Rational Zero Test (Rational Roots Theorem). You do not have to determine which of these candidates are, in fact, zeros. (7 points)

$f(x)$ is a polynomial with integer coefficients and GCF = 1, so the theorem applies.

p: $\pm 1, \pm 3$ (← Factors of -3, the constant term)

q: $\pm 1, \pm 2$ (← Factors of 2, the leading coefficient)

$\frac{p}{q}$: $\frac{\pm 1, \pm 3}{\pm 1, \pm 2}$ (← Informal)

$\frac{p}{q}$: $\boxed{\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}}$ (← The candidates)

9) Simplify i^{502}. (3 points)

When we divide 502 by 4, the remainder is 2. Therefore, $i^{502} = i^2 = \boxed{-1}$.
(500 is a nice multiple of 4; 502 is 2 more than that.)

10) Let $f(x) = x^5 + 9x^3$. Write the three distinct complex zeros of f and their multiplicities in the table below. (9 points)

Zero	Multiplicity
0	3
$-3i$	1
$3i$	1

Factor: $x^5 + 9x^3 = x^3(x^2 + 9) = x^3(x + 3i)(x - 3i)$, or $(x - 0)^3(x - (-3i))^1(x - 3i)^1$

11) Consider $f(x) = 3x^5 + 2x^4 - x^2 + 3x - 1$. Using only Descartes's Rule of Signs, … (8 points total)

a) List the possible numbers of **positive** real zeros of f (accounting for multiplicity: double roots are counted twice, for example). (3 points)

$f(x) = +3x^5 + 2x^4 - x^2 + 3x - 1$ has 3 variations in sign.

Counting down by twos, there are either 3 or 1 positive real zeros.
Note: There is, in fact, just one: it's about 0.359266 (with multiplicity 1).

b) List the possible numbers of **negative** real zeros of f (accounting for multiplicity: double roots are counted twice, for example). (5 points)

$$f(-x) = 3(-x)^5 + 2(-x)^4 - (-x)^2 + 3(-x) - 1$$
$$= -3x^5 + 2x^4 - x^2 - 3x - 1$$

has 2 variations in sign.

Counting down by twos, there are either 2 or 0 negative real zeros.
Note: There are, in fact, none.

12) Write the equation of the **horizontal** asymptote (HA) for the graph of $y = \dfrac{5x^4 + 3x}{9x^4 - x^2 + 1}$ in the usual xy-plane. (3 points)

$\boxed{y = \frac{5}{9}}$, the **ratio of the leading coefficients** of the numerator and the denominator.
This rule applies because the numerator and denominator have the **same degree** (4).

13) Yes or No: Does the graph of $y = \dfrac{x^2 - 36}{x - 6}$ have a vertical asymptote (VA) with equation $x = 6$ in the usual *xy*-plane? Box in one: (2 points)

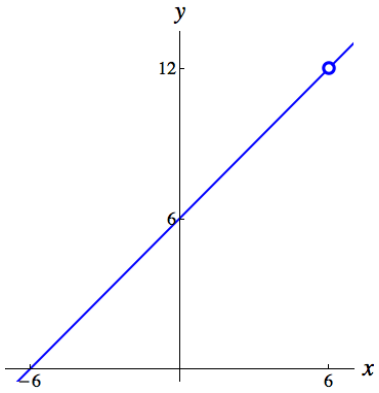
Yes $\qquad$ **No**

When we simplify, all of the $(x-6)$ factors in the denominator are eliminated.

$$\frac{x^2-36}{x-6} = \frac{(x+6)\overbrace{(x-6)}^{(1)}}{\underbrace{x-6}_{(1)}} = x+6 \ (x \neq 6)$$

There is actually a **hole** at the point $(6, 12)$. In Calculus, you will see that:

$\lim\limits_{x \to 6} \dfrac{x^2-36}{x-6} = \lim\limits_{x \to 6} (x+6) = 6+6 = 12$, the ***y*-coordinate** of the hole.



14) Consider the graph of $y = \dfrac{x-4}{x^2-2x-3}$ in the usual *xy*-plane. If an answer to a part below is none, write "NONE." Box in the answers! (14 points total)

Let $f(x) = \dfrac{x-4}{x^2-2x-3}$ (rational with non-0 polynomial numerator and denominator).

a) Find the equation(s) of the vertical asymptote(s) (VAs), if any. (5 points)

Find the **real zeros of the denominator**:

$$x^2-2x-3=0 \Leftrightarrow (x+1)(x-3)=0 \Leftrightarrow [(x=-1) \text{ or } (x=3)]$$

Neither -1 nor 3 is a zero of the numerator. The given fraction is **simplified**.

The **VAs** have equations: $\boxed{x=-1 \text{ and } x=3}$.

b) Find the equation of the horizontal asymptote (HA), if any. (3 points)

$\boxed{y=0}$, because the **degree of the numerator** of $\dfrac{x-4}{x^2-2x-3}$ is 1, which is **less than** the **degree of the denominator**, which is 2. The given rational expression is **proper ("bottom-heavy")**, so the x-axis is an **HA**.

c) Find the x-intercept(s), if any. (3 points)

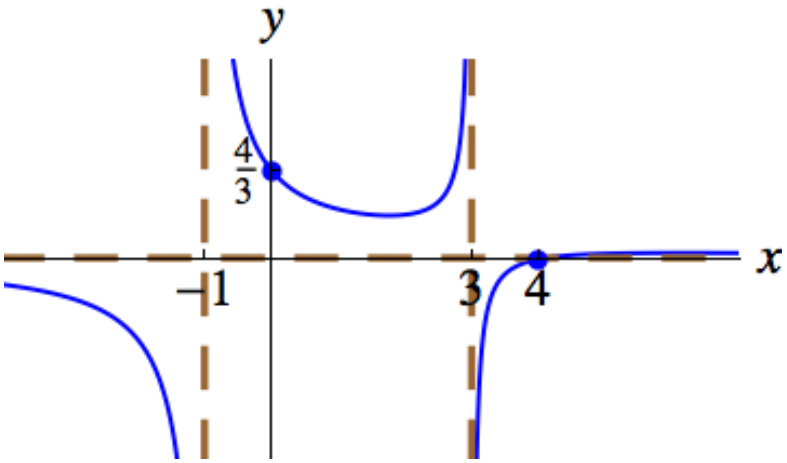
Find the **real zeros** of f by setting y or $f(x)=0$ and solving for x.

$$0=\frac{x-4}{x^2-2x-3} \Leftrightarrow 0=x-4 \ (x\neq -1 \text{ and } x\neq 3) \Leftrightarrow x=4$$

The only ***x*-intercept** is at: $\boxed{(4,0)}$.

d) Find the y-intercept, if any. (3 points)

Set $x=0$. $f(0)=\dfrac{(0)-4}{(0)^2-2(0)-3}=\dfrac{-4}{-3}=\dfrac{4}{3} \Rightarrow \boxed{\left(0,\frac{4}{3}\right)}$ is the ***y*-intercept**.



15) Write the domain of f, where $f(x)=\sqrt{x^2+x-6}$, using interval form (the form using parentheses and/or brackets). (6 points)

• We have an even-root radical (the index, 2, is even), so we require that the radicand be nonnegative: $f(x)$ is real $\Leftrightarrow$ $x^2+x-6\geq 0$.

• See the Notes for Section 2.7 on $f(x)=\sqrt{x^2-9}$.

• The key differences are that x^2+x-6 factors as $(x+3)(x-2)$, it has -3 and 2 as zeros, and $\text{Dom}(f)=\boxed{(-\infty,-3]\cup[2,\infty)}$. See the graph of $y=x^2+x-6$ above.

16) Let $f(x) = e^x \ln(x+5)$. Write the **domain** of f using interval form (the form using parentheses and/or brackets). (3 points)

The domain of e^x is $\mathbb{R}$. $\ln(x+5)$ is real $\Leftrightarrow$ $x+5>0$ $\Leftrightarrow$ $x>-5$, so the domain of $\ln(x+5)$, and of f, is: $\boxed{(-5,\infty)}$.

17) Simplify the following: (6 points total; 2 points each)

a) $\log_8\left(\frac{1}{64}\right)$

$\log_8\left(\frac{1}{64}\right) = \boxed{-2}$. This is because: $8^{\boxed{-2}} = \frac{1}{8^2} = \frac{1}{64}$.

b) $\log_7\left(7^{10}\right)$

$\log_7\left(7^{10}\right) = \boxed{10}$ (by Inverse Properties). Think: $7^{\boxed{10}} = 7^{10}$

c) $\log_{25}(5)$

$\log_{25}(5) = \boxed{\frac{1}{2}}$. This is because: $25^{\boxed{1/2}} = \sqrt{25} = 5$.

18) Expand and evaluate where appropriate: $\log\left[\dfrac{x^2\left(\sqrt[3]{y}\right)}{1000z^3}\right]$. Assume $x, y, z > 0$.

(10 points)

$$\log\left[\frac{x^2\left(\sqrt[3]{y}\right)}{1000z^3}\right] = \left[\log\left(x^2\right)+\log\left(\sqrt[3]{y}\right)\right] - \left[\log(1000)+\log\left(z^3\right)\right]$$

[by the Product and Quotient Rules for Logarithms; watch grouping symbols!]

$$= \log\left(x^2\right) + \log\left(\sqrt[3]{y}\right) - \log(1000) - \log\left(z^3\right)$$

[Note the last "−."]

$$= \log\left(x^2\right) + \log\left(y^{1/3}\right) - 3 - \log\left(z^3\right)$$

[Observe: $\log(1000) = \log_{10}\left(10^3\right) = 3$]

$$= \boxed{2\log(x) + \frac{1}{3}\log(y) - 3 - 3\log(z)}$$

[by the Power / Smackdown Rule for Logarithms]

Note: If we had allowed $x < 0$, then we would need $2\log|x|$ instead of $2\log(x)$.

19) Find all real solution(s) of the equation: $\log_2(x)+\log_2(x-1)=1$. Write the solution set. Show all work, as in class; do not use trial-and-error! (9 points)

Note: We will consider domain issues at the end, when we check.

$$\log_2(x)+\log_2(x-1)=1$$
$$\log_2\left[x(x-1)\right]=1 \qquad \text{[by Product Rule for Logs]}$$
$$\log_2\left[x^2-x\right]=1$$
$$2^1=x^2-x \qquad \text{[Go to Exponential Form; see Note (*).]}$$
$$2=x^2-x$$
$$0=x^2-x-2$$
$$0=(x-2)(x+1)$$
$$x=2 \text{ or } \cancel{x=-1}$$

Checking tentative solutions is required when solving log equations.

$x=-1$ does **not** check out; $\log_2(-1)$ and $\log_2(-2)$ are **not real**.

Check $x=2$:	Check $x=-1$:
$\log_2(2)+\log_2(2-1)=1$ $1+\log_2(1)=1$ $1+0=1$ $1=1$ (Checks out.)	$\log_2(-1)+\log_2(-1-1)=1$ $\log_2(-1)+\log_2(-2)=1$ (Check fails.)

The solution set is: $\boxed{\{2\}}$.

Note (*): We could have performed base-2 exponentiation on both sides:

$$2^{\log_2\left[x^2-x\right]}=2^1$$
$$x^2-x=2$$

20) Find all real solution(s) of the equation: $e^{2x}-7e^x=0$.
Write the solution set. Show all work! Hint: Factor. (7 points)

$$e^{2x}-7e^x=0, \quad \text{or} \quad u^2-7u=0 \ (\text{where } u=e^x)$$
$$e^x(e^x-7)=0, \quad \text{or} \quad u(u-7)=0 \ (\Leftrightarrow u=0 \text{ or } u-7=0)$$

$\cancel{e^x=0}$

$(0 \text{ is not in the range of } e^x.)$

or

$$e^x-7=0$$
$$e^x=7$$
$$\ln(e^x)=\ln(7)$$
$$x=\ln(7)$$

The solution set is: $\boxed{\{\ln(7)\}}$.